

GENIUS IS A THING THAT HAPPENS

I have made much of Hilbert's role, as is right, but there's a risk that by paying so much attention to the names at the top of the marquee I'll give a misimpression of mathematics as an enterprise in which a few solitary geniuses, marked at birth, blaze a path for the rest of humankind to trot along. It's easy to tell the story that way. In some cases, like that of Srinivasa Ramanujan, it's not so far off. Ramanujan was a prodigy from southern India who, from childhood, produced astonishingly original mathematical ideas, which he described as divine revelations from the goddess Namagiri. He worked for years completely in isolation from the main body of mathematics, with access to only a few books to acquaint him with the contemporary state of the subject. By 1913, when he finally made contact with the greater world of number theory, he had filled a series of notebooks with something like four thousand theorems, many of which are still the subject of active investigation today. (The goddess provided Ramanujan with theorem statements, but no proofs—those are for us, Ramanujan's successors, to fill in.)

But Ramanujan is an outlier, whose story is so often told precisely because it's so uncharacteristic. Hilbert started out a very good but not exceptional student, by no means the brightest young mathematician in Königsberg; that was Hermann Minkowski, two years younger. Minkowski went on to a distinguished mathematical career himself, but he was no Hilbert.

One of the most painful parts of teaching mathematics is seeing students damaged by the cult of the genius. The genius cult tells students it's not worth doing mathematics unless you're the *best* at mathematics, because those special few are the only ones whose contributions matter. We don't treat any other subject that way! I've never heard a student say, "I like *Hamlet*, but I don't really belong in AP English—that kid who sits

in the front row knows *all* the plays, and he started reading Shakespeare when he was nine!" Athletes don't quit their sport just because one of their teammates outshines them. And yet I see promising young mathematicians quit every year, even though they love mathematics, because someone in their range of vision was "ahead" of them.

We lose a lot of math majors this way. Thus, we lose a lot of future mathematicians; but that's not the whole of the problem. I think we need more math majors who *don't* become mathematicians. More math major doctors, more math major high school teachers, more math major CEOs, more math major senators. But we won't get there until we dump the stereotype that math is only worthwhile for kid geniuses.

The cult of the genius also tends to undervalue hard work. When I was starting out, I thought "hardworking" was a kind of veiled insult—something to say about a student when you can't honestly say they're smart. But the ability to work hard—to keep one's whole attention and energy focused on a problem, systematically turning it over and over and pushing at everything that looks like a crack, despite the lack of outward signs of progress—is not a skill everybody has. Psychologists nowadays call it "grit," and it's impossible to do math without it. It's easy to lose sight of the importance of work, because mathematical inspiration, when it finally does come, can feel effortless and instant. I remember the first theorem I ever proved; I was in college, working on my senior thesis, and I was completely stuck. One night I was at an editorial meeting of the campus literary magazine, drinking red wine and participating fitfully in the discussion of a somewhat boring short story, when all at once something turned over in my mind and I understood how to get past the block. No details, but it didn't matter; there was no doubt in my mind that the thing was done.

That's the way mathematical creation often presents itself. Here's the French mathematician Henri Poincaré's famous account of a geometric breakthrough he made in 1881:

Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step the idea came to me, without anything in my former thoughts seeming to have

paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidean geometry. I did not verify the idea; I should not have had time, as, upon taking my seat in the omnibus, I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience's sake I verified the result at my leisure.*

But it didn't *really* happen in the space of a footstep, Poincaré explains. That moment of inspiration is the product of weeks of work, both conscious and unconscious, which somehow prepare the mind to make the necessary connection of ideas. Sitting around waiting for inspiration leads to failure, no matter how much of a whiz kid you are.

It can be hard for me to make this case, because I was one of those prodigious kids myself. I knew I was going to be a mathematician when I was six years old. I took courses way above my grade level and won a neckful of medals in math contests. And I was pretty sure, when I went off to college, that the competitors I knew from Math Olympiad were the great mathematicians of my generation. It didn't exactly turn out that way. That group of young stars produced many excellent mathematicians, like Terry Tao, the Fields Medal-winning harmonic analyst. But most of the mathematicians I work with now weren't ace mathletes at thirteen; they developed their abilities and talents on a different time-scale. Should they have given up in middle school?

What you learn after a long time in math—and I think the lesson applies much more broadly—is that there's *always* somebody ahead of you, whether they're right there in class with you or not. People just starting out look to people with good theorems, people with some good theorems look to people with lots of good theorems, people with lots of good theorems look to people with Fields Medals, people with Fields Medals look to the “inner circle” Medalists, and those people can always look toward the dead. Nobody ever looks in the mirror and says, “Let's face it, I'm smarter than Gauss.” And yet, in the last hundred

* From Poincaré's essay “Mathematical Creation,” highly recommended reading if you care about mathematical creativity, or for that matter any kind of creativity.

years, the joined effort of all these dummies-compared-to-Gauss has produced the greatest flowering of mathematical knowledge the world has ever seen.

Mathematics, mostly, is a communal enterprise, each advance the product of a huge network of minds working toward a common purpose, even if we accord special honor to the person who places the last stone in the arch. Mark Twain is good on this: "It takes a thousand men to invent a telegraph, or a steam engine, or a phonograph, or a telephone or any other important thing—and the last man gets the credit and we forget the others."

It's something like football. There are moments, of course, when one player seizes control of the game totally, and these are moments we remember and honor and recount for a long time afterward. But they're not the normal mode of football, and they're not the way most games are won. When the quarterback completes a dazzling touchdown pass to a streaking wide receiver, you are seeing the work of many people in concert: not only the quarterback and the receiver, but the offensive linemen who prevented the defense from breaking through just long enough to allow the quarterback to set and throw, that prevention in turn enabled by the running back who pretended to take a handoff in order to distract the attention of the defenders for a critical moment; and then, too, there's the offensive coordinator who called the play, and his many clipboarded assistants, and the training staff who keep the players in condition to run and throw . . . One doesn't call all those people geniuses. But they create the conditions under which genius can take place.

Terry Tao writes:

The popular image of the lone (and possibly slightly mad) genius—who ignores the literature and other conventional wisdom and manages by some inexplicable inspiration (enhanced, perhaps, with a liberal dash of suffering) to come up with a breathtakingly original solution to a problem that confounded all the experts—is a charming and romantic image, but also a wildly inaccurate one, at least in the world of modern mathematics. We do have spectacular, deep

and remarkable results and insights in this subject, of course, but they are the hard-won and cumulative achievement of years, decades, or even centuries of steady work and progress of many good and great mathematicians; the advance from one stage of understanding to the next can be highly non-trivial, and sometimes rather unexpected, but still builds upon the foundation of earlier work rather than starting totally anew. . . . Actually, I find the reality of mathematical research today—in which progress is obtained naturally and cumulatively as a consequence of hard work, directed by intuition, literature, and a bit of luck—to be far more satisfying than the romantic image that I had as a student of mathematics being advanced primarily by the mystic inspirations of some rare breed of “geniuses.”

It's not wrong to say Hilbert was a genius. But it's more right to say that what Hilbert accomplished was genius. Genius is a thing that happens, not a kind of person.