Cuckoo Filter: Simplification and Analysis

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Context

Goal: Data structure for a set of n identifiers (keys) drawn from a larger universe of U potential identifiers

Want fast membership queries, small memory footprint

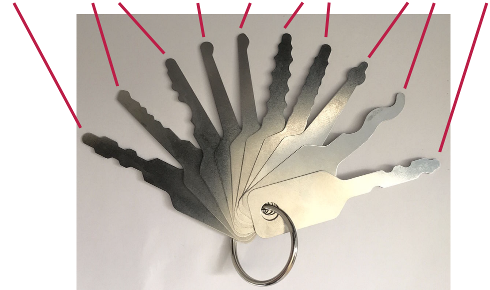
Other options (insert, delete, union, intersect) also useful

File:Wafer Lock Try-Out Keys.jpg by Willh26 on Wikimedia commons

Exact solutions: Bit vector

Store an array of bits, one per possible key
1 for set members, 0 for nonmembers

1	0	0	1	1	0	0	1	0	1	0	1	1	0	0	1	1	0	1	0



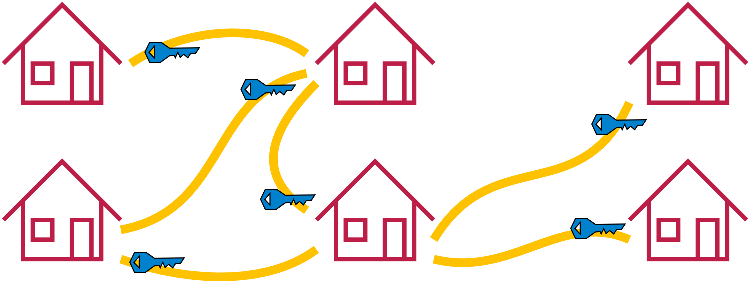
Fast queries, and vectorized union and intersection operations
But memory requirement $\Theta(U)$ is too large

Exact solutions: Cuckoo hashing (I)

[Pagh and Rodler 2004]

Each key is hashed to two home locations

Assign keys to homes and store one key per home



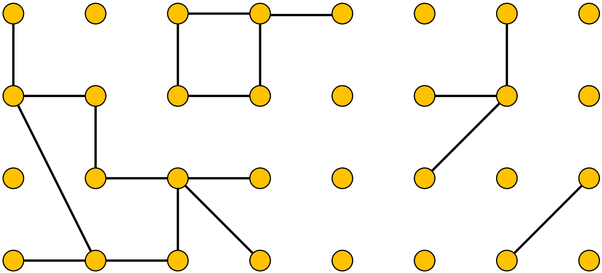
Constant worst-case query time (check both locations)

Constant average-case updates

Failure (unable to match keys to homes) has probability $O(1/n)$

Exact solutions: Cuckoo hashing (II)

Succeeds in matching keys to homes $\Longleftrightarrow$ the graph (homes, pairs selected by keys) is a *pseudoforest* (each component has ≤ 1 cycle)



Two weaknesses:

Failure probability of $O(1/n)$ may be too high

To achieve this, must leave $> 1/2$ of the homes empty (too much wasted memory)

Exact solutions: Blocked cuckoo hashing

Store multiple keys/location [Dietzfelbinger and Weidling 2007]

Succeeds when no subset of location has too many keys

Allows near-optimal space $(1+\epsilon)n\log_2 U$

Improves failure probability to 1/polynomial [Kirsch et al. 2010]

When even optimal space is too much

Reasons to use very little memory:

- Huge data sets,
  too large to fit into main memory
- Small embedded devices
  with little available memory
- Performance from fitting in cache

Solution: Approximate data structures!

Less memory but imprecise answers

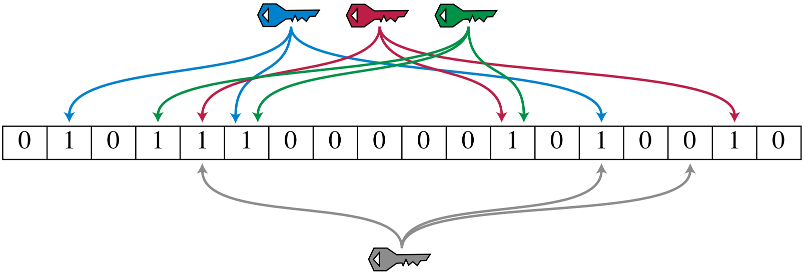
File:4856 - VIC-1211A Super Expander w 3k RAM open.JPG by Sven.petersen on Wikimedia commons

Approximate solutions: Bloom filter

[Bloom 1970]

Uses bitvector idea, but hashes each key to $O(1)$ bitvector cells

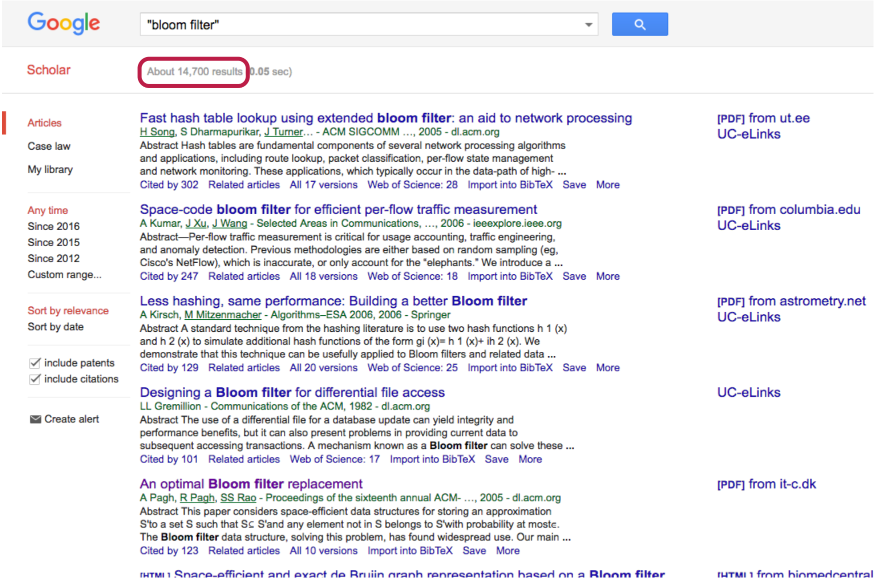
Query answer true $\Longleftrightarrow$ all hashed cells nonzero



A small number of keys that are not in the set will also have all cells nonzero – **false positives**

Uses $O(n \log 1/\rho)$ bits for false positive rate ρ

Bloom filters: enormously popular in practice



Drawbacks of Bloom filters

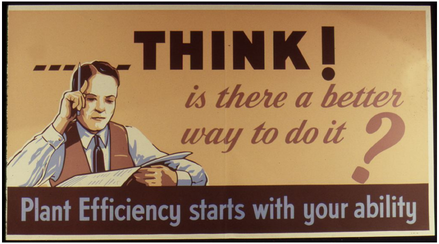
- Suboptimal memory
  44% worse than lower bound
- Unable to delete items
  (counting Bloom filter can but uses $\omega(1)$ more memory)
- Poor memory access pattern
  More accurate $\Rightarrow$ more hits/query

File:2008 08 19 Einbreid Bru Iceland.JPG by Crux on Wikimedia commons

Better than Bloom filters

"An optimal Bloom filter replacement" [Pagh et al. 2005]

"Cuckoo filter: Practically better than Bloom" [Fan et al. 2014]



Both have optimal space, locality of reference, allow deletions

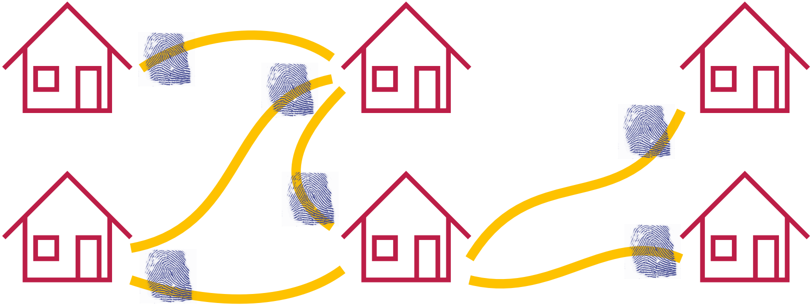
Pagh et al.: proven, but no practical implementation

Fan et al.: practical implementation but no proofs

...until now

Cuckoo filter main idea

Cuckoo hash, but save space by storing fingerprints instead of keys

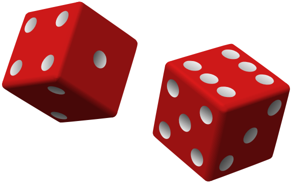


Based on File:Ninhydrin staining thumbprint.png by Horoporo on Wikimedia commons

Answer query by checking whether the query key's fingerprint is at one of its homes

Complication: How to reshuffle keys after an insert?

In cuckoo hashing, homes are *independent* functions of key



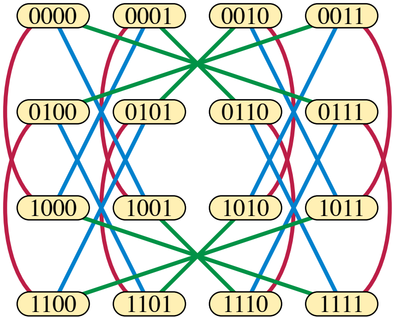
But cuckoo filter reshuffle only knows fingerprint+location, not key

Not enough information for second home to be independent

Solution: use hash(key) and hash(key) xor hash(fingerprint)

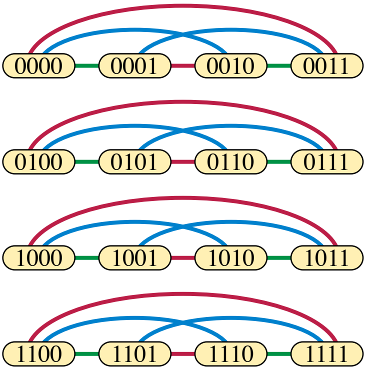
Simplification: hash(key) and hash(key) xor fingerprint

Graph of pairs of homes for all fingerprints



Second home = first home xor hash(fingerprint)

Colors show different hash values

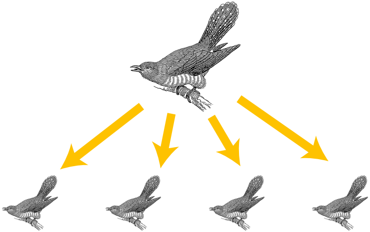


Second home = first home xor fingerprint

Colors show different (2-bit) fingerprints

Main ideas of analysis

When we use simplified home placement,
we are effectively partitioning the cuckoo filter
into many smaller cuckoo filters



The partition is highly likely to be well balanced
(standard argument using Chernoff bounds)

Within each of the smaller cuckoo filters,
pairs of homes are independent of each other
so we can use existing cuckoo hash analysis

Conclusions

The simplified cuckoo filter with sufficiently large constant b fingerprints/home and fingerprint size $f = \Omega((\log n)/b)$ can place all fingerprints with high probability

When it succeeds, it achieves false positive rate $\rho = O(b/2^f)$ using memory arbitrarily close to optimal, $(1+\epsilon)n \log_2 1/\rho$ bits

File:Success sign.jpg by rmgimages from Wikimedia commons

Still open: Analyze cuckoo filtering without the simplification

References I

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References II

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