Approximating Spatial Evolutionary Games using Bayesian Networks

Extended Abstract

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ABSTRACT

Evolutionary Game Theory is an application of game theory to evolving populations of organisms. Of recent interest are EGT models situated on structured populations or spatial evolutionary games. Due to the complexity added by introducing a population structure, model analysis is usually performed through agent-based Monte-Carlo simulations. However, it can be difficult to obtain desired quantities of interest from these simulations due to stochastic effects. We define a framework for modeling spatial evolutionary games using Dynamic Bayesian Networks that capture the underlying stochastic process. The resulting Dynamic Bayesian Networks can be queried for quantities of interest by performing exact inference on the network. We then propose a method for producing approximations of the spatial evolutionary game through the truncation of the corresponding DBN, taking advantage of the high symmetry of the model. This method generalizes mean-field and pair approximations in the literature for spatial evolutionary games. Furthermore, we show empirical results demonstrating the capability of the method to obtain much better accuracy than pair approximation with respect to stochastic simulations.

KEYWORDS

Spatial Evolutionary Games, Bayesian Networks, Moment Closure

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1 INTRODUCTION

Evolutionary Game Theory (EGT) was initially developed to model biological evolution [19] but has found additional use in research on the evolution of cultural phenomena [6], and a variety of multi-agent systems topics [2, 23, 26, 27, 31]. While far from an exact description of human interactions, EGT models can be used to find trends that capture essential characteristics of modeled interactions [5, 7]. One can rely on agent-based stochastic simulations [1] to obtain insights. However, these simulations come with their own limitations in validation [18] and variability [21]. A commonly used alternative is pair approximation [13, 17] which has been used to obtain qualitative insight into EGT models [12, 13, 20, 24] even if is not very accurate to the underlying stochastic model [15, 16, 25, 29].

We propose a framework for exact modeling of spatial evolutionary games using a Dynamic Bayesian Network (DBN) [4], thus making the whole toolbox of probabilistic inference algorithms applicable to such stochastic games [3, 8–11, 14, 22]. Then we develop a method for producing approximations of stochastic spatial evolutionary games, by exploiting symmetry, through the truncation of the corresponding DBN. This provides a flexible framework for the exploration of higher order approximations beyond pair approximation that allow for better accuracy with respect to the underlying stochastic model. Finally, we provide preliminary empirical results illustrating the potential of our approach in modeling stochastic simulations and its advantages over existing approximations.

2 DBN EVOLUTIONARY-GAME MODEL

We consider a population of M agents $\{1, ..., M\}$ placed on evenly-spaced points in a grid with circulatory boundary conditions. An evolutionary game consists of T iterations, each having an interaction phase followed by an update phase. In the interaction phase, each agent i chooses some action $s_i \in S$ and receives a payoff π_i as the sum of the payoffs received from playing a normal-form game with payoff matrix $\mathbf{U}$ against each of its neighbors $N(i)$. Each time the update phase occurs, a percentage of agents γ in the population use an update rule to decide how to change their strategies. For example, in the Fermi rule agents choose a random neighbor to compare their payoff with. Agents may also have a probability μ of mutating to a random strategy during the update phase [30].

We next define a model that fully captures our spatial evolutionary game using a Dynamic Bayesian Network (DBN). The DBN is $(X(t), D(t), P(t))$. The variable set $X(t) = X$ is split into two sets of variables $X = A \cup Pay$, where

- $A_{i,j}(t) \in A$: the strategy of the agent at coordinate (i, j) on the grid at the start of each iteration t.
- $Pay_{i,j}(t) \in Pay$: the payoff received by the agent at (i, j) during the interaction phase at time t.

CPT for Payoff Nodes Each node $Pay_{i,j}(t)$ has $d + 1$ parents consisting of $A_{i,j}(t)$ and its d neighbors $N(A_{i,j}) = \{A_{k,l} | (k, l) \in N(i, j)\}$. The conditional probability function $P(Pay)_{i,j}(t) \mid parents)$ is constructed from the payoff matrix $\mathbf{U}$, expressed as a CPT:

$$\Pr(Pay_{i,j}(t) \mid A_{i,j}(t), N(A_{i,j}(t)))$$
$$= \begin{cases} 1 & \text{if } Pay_{i,j}(t) = \sum_{(k,l) \in N(i,j)} \mathbf{U}(A_{i,j}(t), A_{k,l}(t)) \\ 0 & \text{otherwise} \end{cases}$$

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CPT for Strategy Variables. Each $A_{i,j}(t{+}1)$ has 2(d+1) parents: $A_{i,j}(t)$, $Pay_{i,j}(t)$ and the $A(t)$ and $Pay(t)$ variables for each of the d neighboring $N(i,j)$ agents. Our goal is to define $\Pr(A_{i,j}(t+1) \mid parents)$. We break up this probability into cases controlled by three variables:

- update: did an update happen (yes, with probability γ)?
- mut: did mutation happen (yes, with probability μ)?
- rand: which neighbor was chosen ((k,l), with probability $\frac{1}{d}$)?

We use indicator functions to define

$$\Pr_{\delta} = \mathbb{1}_{A_{i,j}(t+1)=A_{k,l}(t)}, \Pr_{\varnothing} = \mathbb{1}_{A_{i,j}(t+1)=A_{i,j}(t)}$$

For example, if (update = 1) and (mut = 0), we can write:

$$\Pr(A(t+1)_{i,j} = s_{t+1} \mid A_{i,j}(t) = s_t, \text{other parents}) =$$

$$\sum_{(k,l)\in N(i,j)} \frac{1}{d} \Pr_u(Pay_{i,j}, Pay_{k,l}) \Pr_{\delta}(1 - \Pr_{\varnothing}) + \Pr_{\varnothing}$$

where $\Pr_u(Pay_{i,j}, Pay_{k,l})$ is the probability that the agent (i,j) switches to strategy of the neighboring agent (k,l).

The resulting DBN formulation fully encodes the stochastic process of the spatial evolutionary game. However, it is well known that exact inference is exponential in the tree-width of the network. To address this computation issue we propose a novel method for truncating the full DBN taking inspiration from moment-closure methods in the mean-field approximation literature.

3 TRUNCATION APPROXIMATION

We construct a Bayesian Network for each iteration that takes the states of each agent from input nodes at time t to output nodes at time $t{+}1$. Since the marginal distributions for each agent node in the exact model are identical, we can exploit symmetry by looking at the distribution of a few representative nodes. The idea is to truncate the DBN around a single focal agent and some of its neighbors. The input nodes consist of agents in the truncation neighborhood and the output nodes will be a few (e.g. one or two) nodes. We then use the transition probabilities in the exact model to link the input nodes to the output nodes. Since there are less output nodes than input nodes at the next time step, we must also approximate the joint distribution of the truncation neighborhood at the start of each iteration. There are three steps:

Truncation Neighborhood First, we choose some subset of agent nodes $B \subset A$ (examples shown in Fig. 2). We construct the Bayesian Network from time t to time $t+1$ for just the nodes in B. On the truncated DBN, we can now run exact inference algorithms in a reasonable amount of time in order to compute the marginal and pair-wise conditional distributions of the output nodes at $t+1$.

Output query Second, we query a selection of lower order distributions from the output nodes at time $t+1$: $P_{s_i}, P_{s_i|s_j}$.

Input definition Finally, we approximate the joint distribution over the truncation neighborhood at the next iteration using a function of the single variable and pair distributions obtained during the output query step. We can approximate this using a tree-like Bayesian Network. We refer to our full paper for details.

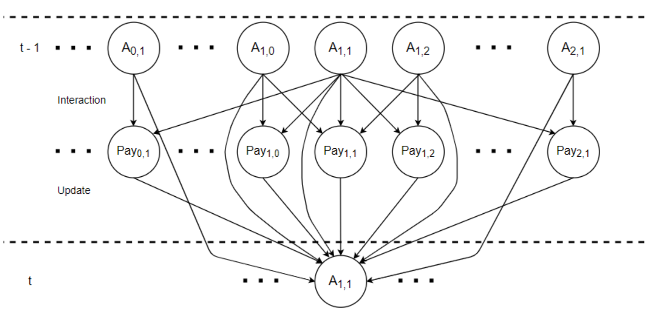

Figure 1: Slice of Dynamic Bayesian Network for the Fermi update rule centered at the agent located at position (1,1)

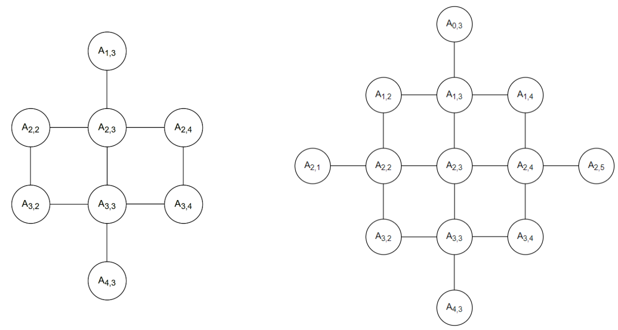

Figure 2: Example Truncation Neighborhoods

4 EMPIRICAL EVALUATION

We run empirical experiments on several games from the evolutionary game theory literature. We test four different cases (BN-MF, BN-PA, BN-Medium, BN-Large) with increasing truncation neighborhoods. Approximation results are compared to the average of 20 simulations on a 50x50 grid. Figure 3 shows two examples.

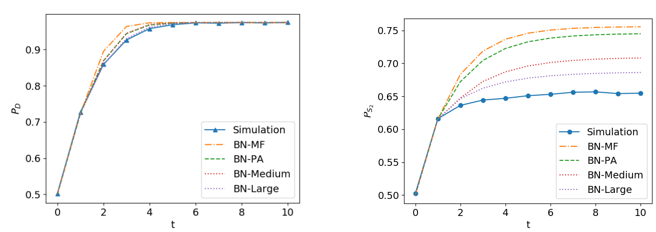

Figure 3: Proportion of agents playing the second strategy in a Prisoner's Dilemma game (left) and a Snowdrift game (right) for different approximations.

Our results show that larger approximation neighborhoods reduce the error in the time evolution graphs, even in games such as Snowdrift (see [28, Section 3.8]) where pair approximation does not have good quantitative agreement with simulation results.

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