Submodel Decomposition Bounds for Influence Diagrams

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Summary

Tree Decomposition for Influence Diagrams and Limited Memory Influence Diagrams

- Graph separation criteria for identifying single-stage decision problem
- Valuation algebra over submodels using graph-based operations
- Submodel-Tree Clustering and Elimination Scheme

Submodel Decomposition Bounds

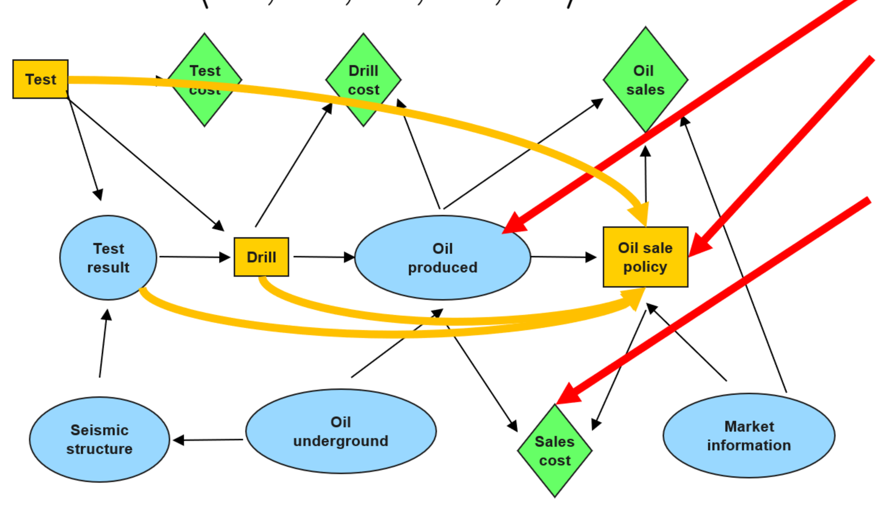
- Bounding MEU by exponentiating utility functions
- Re-use decomposition bounds in Marginal MAP inference

Contributions

- Generate submodel-tree with lower tree-width by removing irrelevant variables and functions in each submodel
- Scalable convex upper bounds for MEU

Influence Diagrams [Howard and Matheson, 1984]

$\mathcal{M} := \langle \mathbf{X}, \mathbf{D}, \mathbf{P}, \mathbf{U}, \mathcal{O} \rangle$



Chance variables $\mathbf{X} = \{X_1, X_2, \dots, X_n\}$

Decision variables $\mathbf{D} = \{D_1, D_2, \dots, D_m\}$

Probability functions $\mathbf{P} = \{P_1, P_2, \dots, P_n\}$

Utility functions $\mathbf{U} = \{U_1, U_2, \dots, U_r\}$

Policy functions $\mathbf{\Delta} = \{\Delta_1, \dots \Delta_m\}$

$\mathcal{O} = \{\mathrm{pa}(D_1) \prec D_1 \prec \dots \prec \mathrm{pa}(D_m) \prec D_m\}$

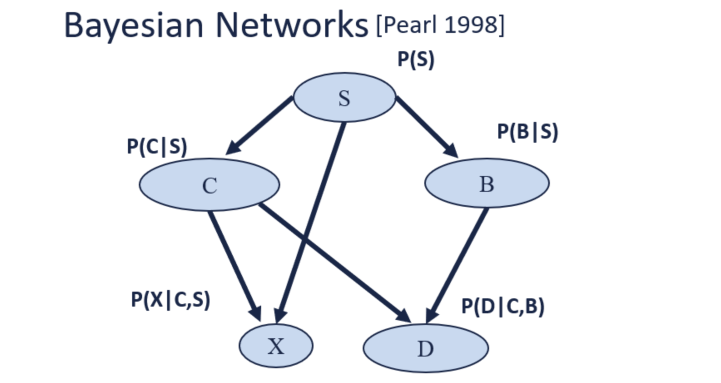
$\Delta_i(D_i | \mathrm{hist}(D_i))$

Maximum expected Utility $\max_{\mathbf{\Delta}} \mathbb{E}_{P(\mathbf{X},\mathbf{D})}[\sum_{U_i \in \mathbf{U}} U_i]$ $\quad P(\mathbf{X},\mathbf{D}) = \prod_{P_i \in \mathbf{P}} P_i \times \prod_{\Delta_i \in \mathbf{\Delta}} \Delta_i$

Optimal strategy $\mathbf{\Delta}^* = \mathrm{argmax}_{\mathbf{\Delta}} \mathbb{E}[\sum_{U_i \in \mathbf{U}} U_i]$

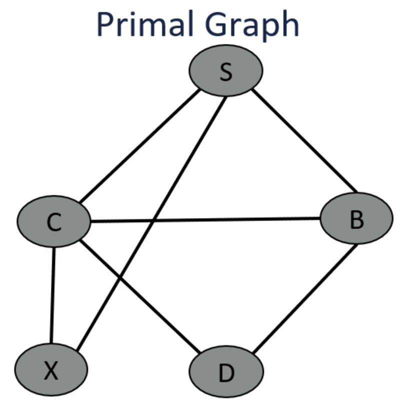
Graphical Models

Bayesian Networks [Pearl 1998]

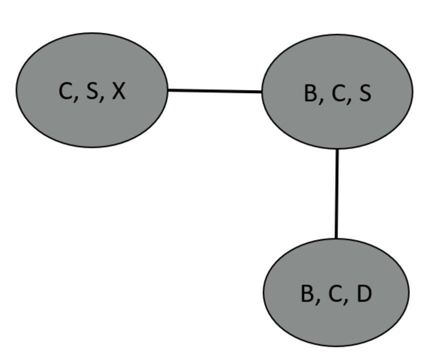


$P(S, C, B, X, D) = P(S)\,P(C|S)\,P(B|S)\,P(X|C,S)\,P(D|C,B)$

Primal Graph

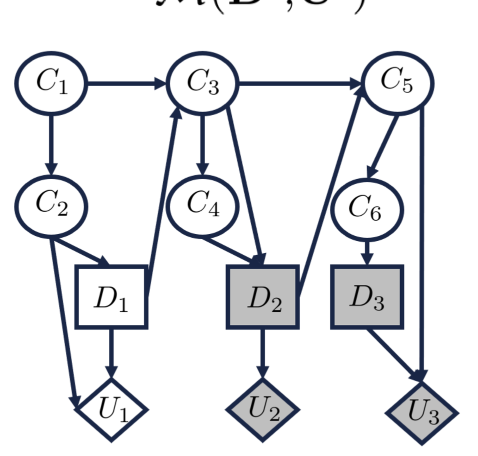


Join-Tree



Partial Evaluation and Local MEU

$$\mathrm{LMEU}_{\mathcal{M}(\mathbf{D}',\mathbf{U}')} := \max_{\mathbf{\Delta}'} \mathbb{E}\big[\textstyle\sum_{U_i \in \mathbf{U}'} U_i | \mathrm{pa}(\mathbf{D}')\big]$$



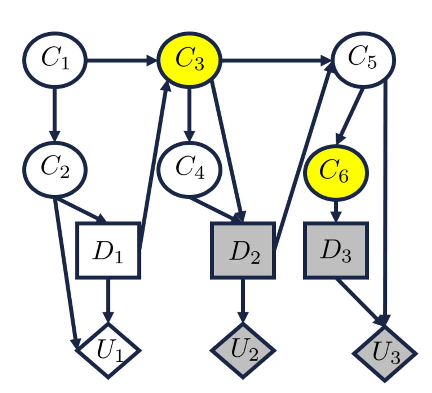
Evaluate MEU over the subset of decision variables and utility functions

- Maximize expected utility U2+U3 over two decision variables D2 and D3

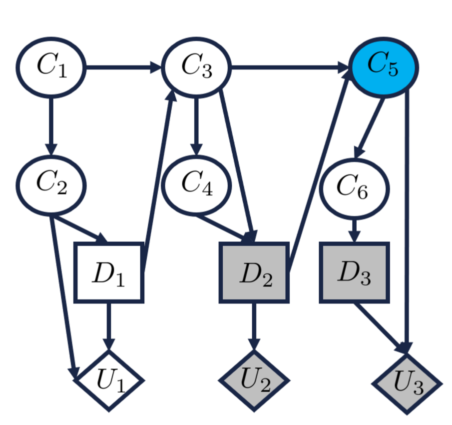
$$\max_{\Delta(D_2|C_3,C_4),\Delta(D_3|C_6)} \sum_{\mathbf{X},\mathbf{D}} \frac{P(\mathbf{X},\mathbf{D})}{P(C_3,C_4,C_6)} [U_2 + U_3]$$

Submodel in IDs

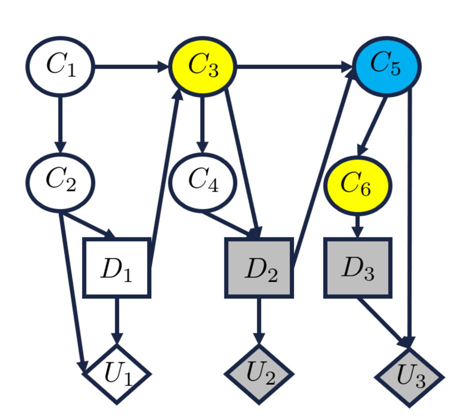
- (Definition) Submodel $\mathcal{M}'(\mathbf{D}',\mathbf{U}')$ is a relevant subset of model $\mathcal{M}$ for computing LMEU on $\mathbf{D}' \subseteq \mathbf{D}, \mathbf{U}' \subseteq \mathbf{U}$



Relevant Observed Variables $\mathrm{REL}_O(\mathbf{D}',\mathbf{U}')$



Relevant Hidden Variables $\mathrm{REL}_H(\mathbf{D}',\mathbf{U}')$



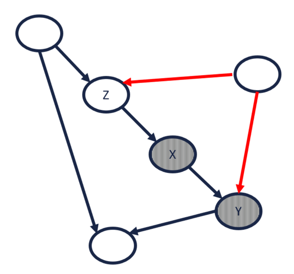
Stable Submodel $\mathcal{M}'(\{D_2, D_3\}, \{U_2, U_3\})$

- (Definition) Submodel $\mathcal{M}'(\mathbf{D}',\mathbf{U}')$ is stable when there is no decision variables in $\mathrm{REL}_H(\mathbf{D}',\mathbf{U}')$

Graph-based Submodel Identification

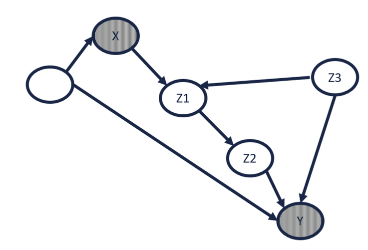
- $\mathrm{REL}_O(\mathbf{D}',\mathbf{U}')$ is the backdoor* set between D' and U'

  (Backdoor) [Pearl 2009]
  a set Z satisfies the backdoor criterion relative to (X, Y)
  (1) None of the nodes in Z is a descendant of X
  (2) Z blocks every path between X and Y that contain arrow into X



- $\mathrm{REL}_H(\mathbf{D}',\mathbf{U}')$ is the union of all frontdoor* set between pa(D') and ch(U')

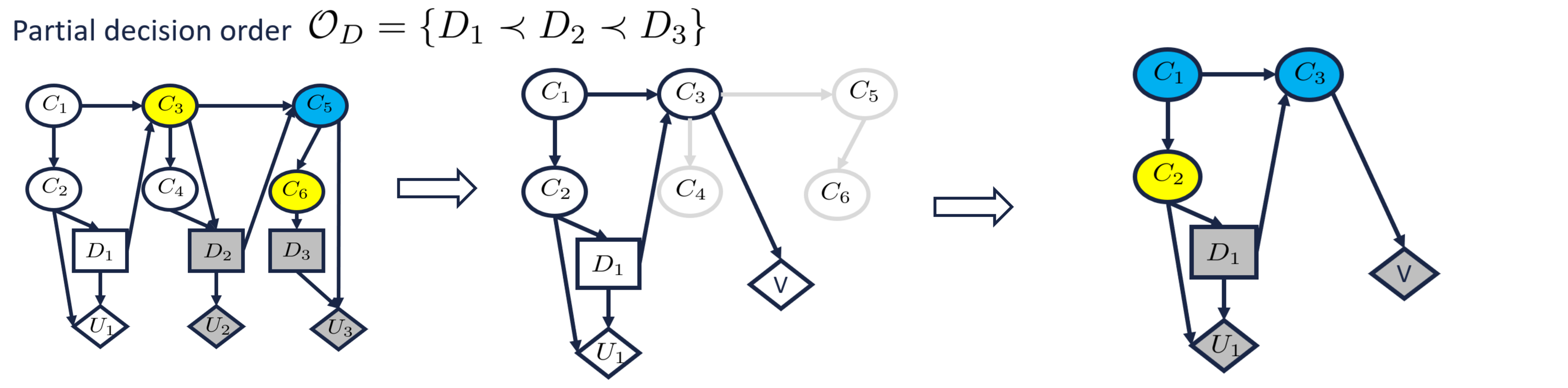
  (Frontdoor) [Pearl 2009]
  a set Z satisfies the frontdoor criterion relative to (X, Y)
  (1) Z intercept all directed paths from X to Y
  (2) There is no backdoor path from X to Z
  (3) All backdoor paths from Z to Y are blocked by X



Submodel-Tree Decomposition

- Process decision nodes in reverse topological order

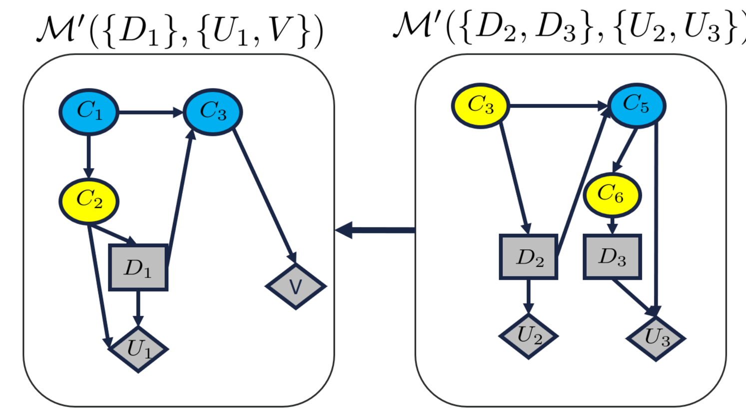
Partial decision order $\mathcal{O}_D = \{D_1 \prec D_2 \prec D_3\}$



Identify 2nd Stable submodel $\quad$ Eliminate submodel from ID $\quad$ Identify 1st Stable Submodel

- Given an ID $\mathcal{M}$, and the set of stable submodels $\mathbf{M}_{\mathcal{O}_D}$ relative to $\mathcal{O}_D$, submodel-tree decomposition is a tuple $\mathcal{T}_{\mathrm{ST}} := \langle T(\mathcal{C},\mathcal{S}), \chi, \psi \rangle$

$\mathcal{M}'(\{D_1\},\{U_1,V\})$ $\quad$ $\mathcal{M}'(\{D_2,D_3\},\{U_2,U_3\})$



$T(\mathcal{C},\mathcal{S})$ Tree of submodel cluster nodes $\mathcal{C}$ and separator edges $\mathcal{S}$

$\chi : \mathcal{C} \to 2^{\mathrm{dom}(\mathcal{M})}$ Label a cluster with a subset of variables in $\mathcal{M}$

$\psi : \mathcal{C} \to 2^{\mathbf{M}_{\mathcal{O}_D}}$ Label a cluster with a subset of submodels in $\mathbf{M}_{\mathcal{O}_D}$

Tree-decomposition satisfies running intersection property

Bounding MEU of Each Submodel

- Exponentiated Utility Bounds for MEU

$$\max_{\mathbf{\Delta}} \mathbb{E}\Big[\sum_{U_i \in \mathbf{U}} U_i(\mathbf{X}_{U_i})\Big] \leq \log \max_{\mathbf{\Delta}} \mathbb{E}\Big[\prod e^{U_i(\mathbf{X}_{U_i})}\Big]$$

LSH: MEU expression with additive utility function
RHS: Upper bound of MEU with log-partition function with exponentiated utility functions

Experiments

Synthetic ID Benchmarks

Domain	n	w_c	w_s	ST-GDD(i=1)	ST-GDD(i=5)	ST-WMB(i=10)	JGDID(i=1)	WMBEID(i=10)
ID-BN	84.6	30.2	21.8	0.19	0.15	**0.13**	0.33	0.74
IDBN14w57d12	115	57	42	103.89	96.24	95.37	1420	2.2E+4
FH-MDP	105.7	25.5	25.4	**0.06**	0.07	0.18	0.16	0.44
mdp9-32-3-8-3	99	43	43	18.92	19.71	25.31	23.09	111.81
FH-POMDP	55.9	28.1	28.1	0.31	0.22	**0.06**	0.56	0.72
pomdp8-14-9-3-12-14	96	47	46	73.53	76.37	67.18	5.E+08	5.E+09
RAND	56.2	20.5	17.9	**0.22**	0.24	0.24	0.23	0.46
rand-c70d21o1	84	32	34	1309.89	1791.93	1752.47	1743.6	2.E+04

- ST-GDD: submodel-tree decomposition + GDD for MMAP [ping et al 2015]
- ST-WMB: submodel-tree decomposition + WMBMMM for MMMAP [marinescu et al 2014]
- JGDID: constrained-join graph + GDD for IDs [Lee et al 2018]
- WMBMEID: constrained mini-bucket tree + WMB/GDD for IDs [Lee et al 2019]

SysAdmin MDP Probabilistic Planning Problem

Instance	c	d	p	u	k	s	w	utime (sec)	ub wmbmm	ltime (sec)	lb sogbofa	gap ($\frac{ub-lb}{ub}$)
mdp10-s50-t3	218	150	218	300	3	11	112	149	162.368	240	142.731	12%
mdp10-s50-t4	274	200	274	400	3	11	112	184	217.515	320	183.650	16%
mdp10-s50-t5	330	250	330	500	3	11	113	257	273.332	400	221.450	19%
mdp10-s50-t6	386	300	386	600	3	11	113	19741	327.268	480	257.394	21%
mdp10-s50-t7	442	350	442	700	3	11	113	28013	383.312	560	291.988	24%
mdp10-s50-t8	498	400	498	800	3	11	113	41748	439.826	640	316.600	28%
mdp10-s50-t9	554	450	554	900	3	11	113	34739	494.662	720	345.844	30%
mdp10-s50-t10	610	500	610	1000	3	11	113	58270	549.867	800	364.569	34%