Submodel Decomposition for Solving Limited Memory Influence Diagrams

Junkyu Lee (junkyul@uci.edu)
University of California, Irvine

Summary

Tree Decomposition for Limited Memory Influence Diagrams

D-separation criteria identify a tree of 1-stage decision problems

Optimal Solution and the Worst-case Complexity

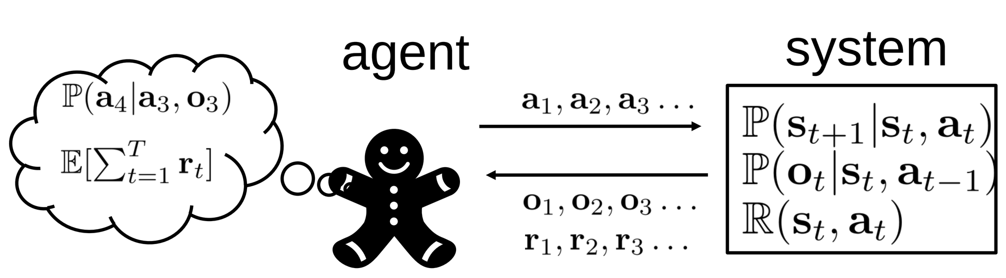
Equilibrium of policy functions within a submodel can be computed by exponential in the treewidth of submodel

Relaxation for Approximate Algorithms

Submodel tree provides a basis of various relaxations

Influence Diagrams[1,2]

Sequential Decision Making Under Uncertainty



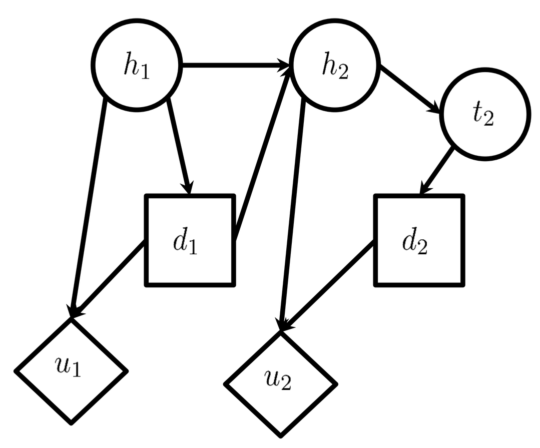
$\mathbb{P}(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ stochastic dynamics over factored state variables

$\mathbb{P}(\mathbf{o}_t|\mathbf{s}_t, \mathbf{a}_{t-1})$ stochastic partial observation

$\mathbb{P}(\mathbf{a}_4|\mathbf{a}_3, \mathbf{o}_3)$ stochastic/non-stationary/ limited memory policy

Graphical Representation of SDM and Optimal Solution

- **Imperfect** recall agent (remember only immediate observations)

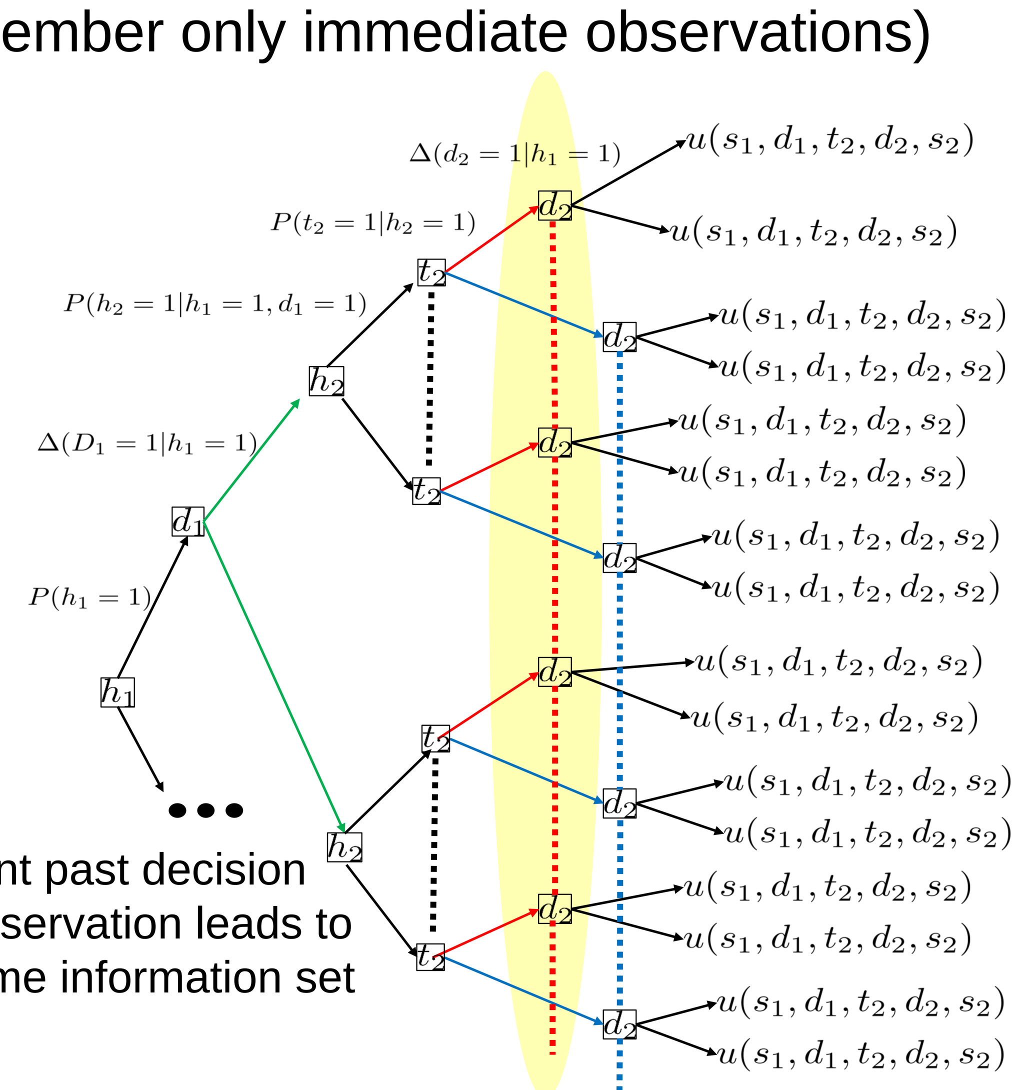


Maximize Expected Utility

$$\max_{\{\Delta_{d_1}, \Delta_{d_2}\}} \mathbb{E}[u_1(h_1, d_1) + u_2(h_2, d_2)]$$

Non-stationary Stochastic Policy

$$\Delta_{d_1}(d_1|h_1) := (h_1, d_1) \rightarrow [0, 1]$$
$$\Delta_{d_2}(d_2|t_2) := (t_2, d_2) \rightarrow [0, 1]$$



Different past decision and observation leads to the same information set

- **Multiselves equilibrium[3,4,5,6] by Bayesian belief update followed by marginalization of missing history**

$$\sum_{h_1} p(h_1) \max_{\Delta_{d_1}} \sum_{d_1} \Delta_{d_1}(d_1|h_1) u_1(h_1, d_1) + V(h_1, d_1)$$

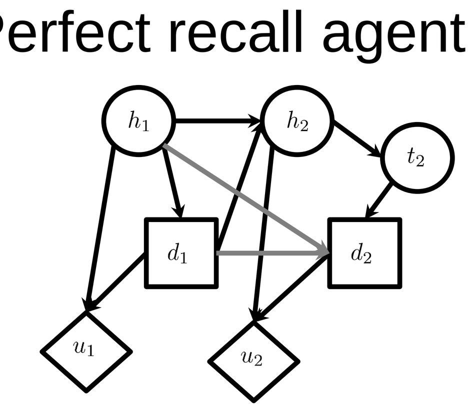
Decision d1

$V(h_1, d_1)$ ← ; → $\Delta_{d_1}(d_1|h_1)$

$$V(h_1, d_1) = \sum_{t_2} \max_{\Delta_{d_2}} \sum_{d_2, h_2} \{p(h2|h_1, d_1) p(t_2|h_2) \times \Delta_{d_2}(d_2|t_2) u_2(h_2, d_2)\}$$

Decision d2

- Perfect recall agent (remember the whole history)

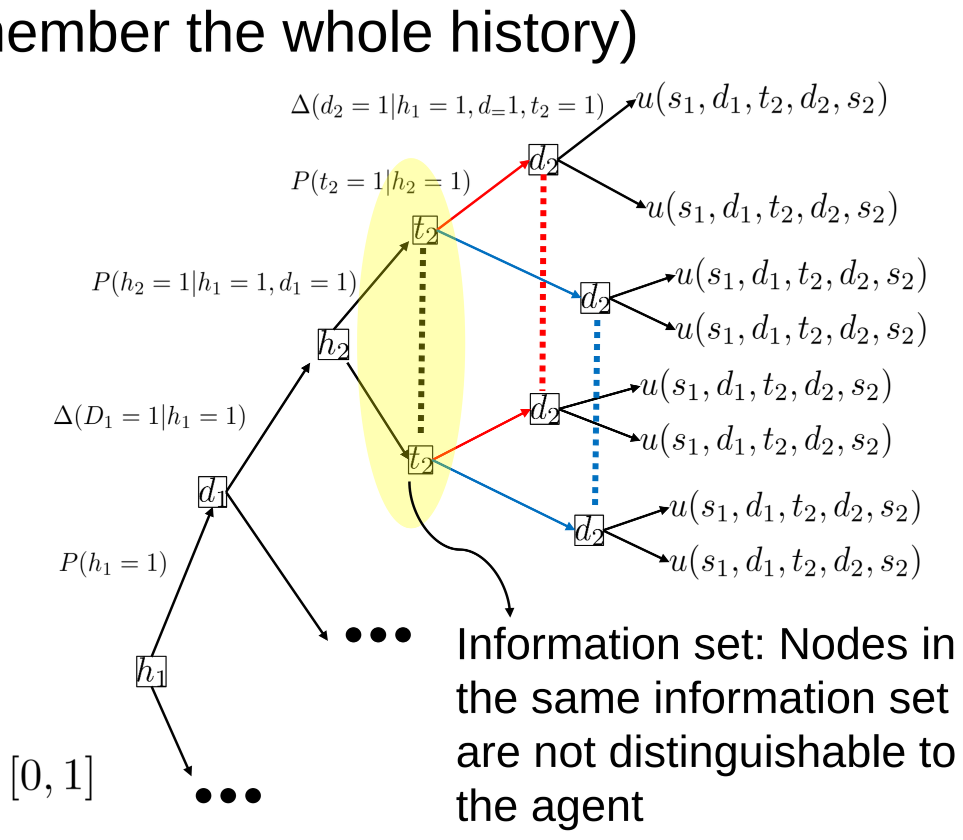


Maximize Expected Utility

$$\max_{\{\Delta_{d_1}, \Delta_{d_2}\}} \mathbb{E}[u_1(h_1, d_1) + u_2(h_2, d_2)]$$

Non-stationary Stochastic Policy

$$\Delta_{d_1}(d_1|h_1) := (h_1, d_1) \rightarrow [0, 1]$$
$$\Delta_{d_2}(d_2|h_1, d_1, t_2) := (h_1, d_1, t_2, d_2) \rightarrow [0, 1]$$



Information set: Nodes in the same information set are not distinguishable to the agent

- **Backward induction with Bayesian Belief Update**

$$\text{MEU} := \sum_{h_1} p(h_1) \max_{\Delta_{d_1}} \sum_{d_1} \Big[\Delta_{d_1}(d_1|h_1) u_1(h_1, d_1) + \Big\{ \sum_{t_2} \max_{\Delta_{d_2}} \sum_{d_2, h_2} p(h2|h_1, d_1) p(t_2|h_2) \Delta_{d_2}(d_2|h_1, d_1, t_2) u_2(h_2, d_2) \Big\} \Big]$$

Decision d1 — Decision d2

Submodel Tree Decomposition

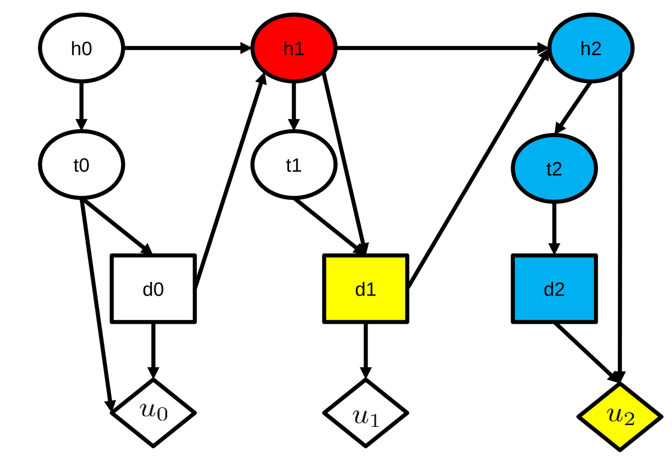
Partial Evaluation and Submodel

- Submodel: Subset of ID relevant for computing partial MEU

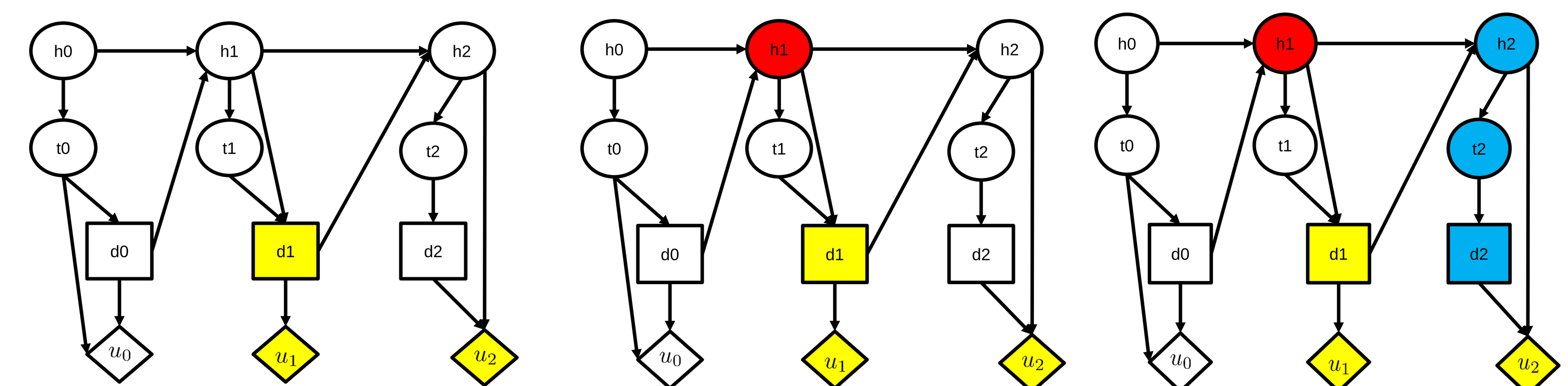
Identify relevant variables and functions for computing expected utility over a subset of decision and utility

$$\max_{\Delta_{d_1}} \mathbb{E}[u_2(h_2, d_2)]$$

observed variables: {h1}
hidden variables: {h2, t2, d2}



Graph Separation Criteria for Identifying Submodel[2,7,8,9]



(a) (b) (c)

(a) $\text{REL}_U(\mathbf{D}) = \text{desc}(\mathbf{D}) \cap \mathbb{U}$

(b) $\text{REL}_O(\mathbf{D}, \mathbf{U}) = \cup_{D \in \mathbf{D}} \text{pa}(D) \cap \text{Backdoor}(\mathbf{D}, \mathbf{U})$

(c) $\text{REL}_H(\mathbf{D}, \mathbf{U}) = \{X | X \in \mathbb{H} \cap \text{Frontdoor}(\cup_{D \in \mathbf{D}} \text{pa}(D), \text{ch}(\mathbf{U}))\}$

Submodel Tree Decomposition $\mathcal{T}$

$\mathcal{O}_T := \{d_T, \cdots, d_1\} \leftarrow$ a total decision order compatible with reverse topological order

$\mathcal{O}_D \leftarrow []$ $\quad \mathbf{X}_D$ = decision variables $\quad \mathbf{D}_{\text{current}} = d_T$

while i in range(T,0, -1) :

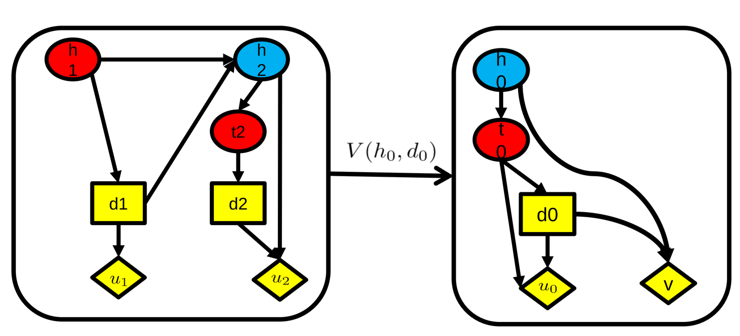
if $\text{REL}_H(\mathbf{D}_{\text{current}}, \text{REL}_U(\mathbf{D}_{\text{current}}))$ has no decision in $\mathbf{X}_D$

$\mathcal{O}_D.append(\mathbf{D}_{\text{current}})$

$\mathbf{X}_D = \mathbf{X}_D \setminus \mathbf{D}_{\text{current}}$

$\mathbf{D}_{\text{current}} = []$

$\mathbf{D}_{\text{current}} = \mathbf{D}_{\text{current}} \cup \{d_i\}$



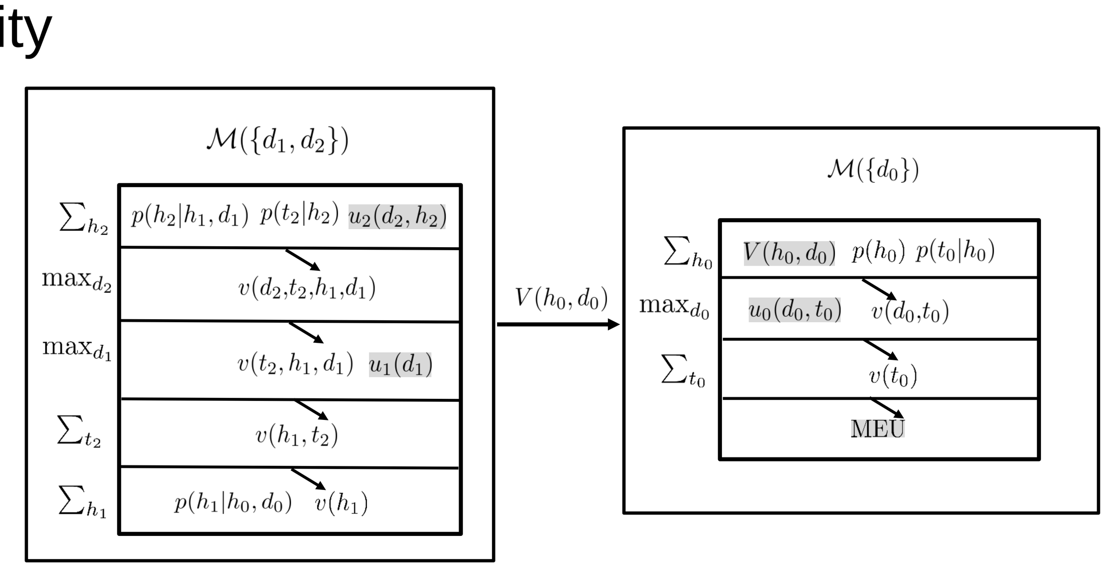
Message Passing (Variable Elimination) over Submodel Tree

- Apply variable elimination[10] for each submodel and propagate conditional expected utility
- Space/time Complexity

$$K^{\max_{\mathcal{M}_i \in \mathcal{T}} w_{\mathcal{M}_i}}$$

K Max. domain size

$w_{\mathcal{M}_i}$ Tree-width of submodel



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