Limited Discrepancy AND/OR Search and its Application to Optimization Tasks in Graphical Models

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Motivation

Graphical Model

- X set of **variables**
- $f_i(X_i)$ set of (*local*, $X_i \subset X$) **cost functions**.

Min-Sum Problem

$$\min_X F(X) = \sum_i f_i(X_i)$$

Applications: Image processing, Natural Language Processing, Bioinformatics, Planning, Resource Allocation, ...

Motivation

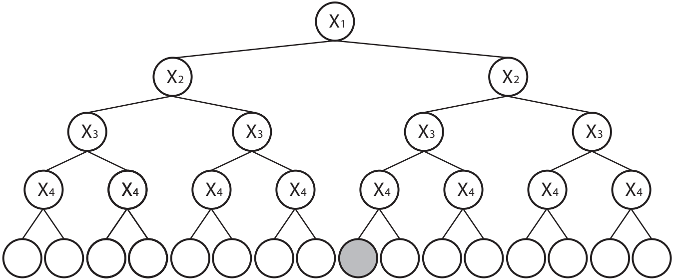
Solving Method: Depth-first Search

Search Space	**OR tree**	**AND/OR tree**	**AND/OR graph**
Size (exp. on)	n. of variables	path width	induced width

- LDS (on the OR tree) very successful **anytime** algorithm (*toulbar2*, *daoopt*)
- Can we adapt LDS to AND/OR search spaces???
- We show that LDSAO is faster than LDS on the min-sum problem.

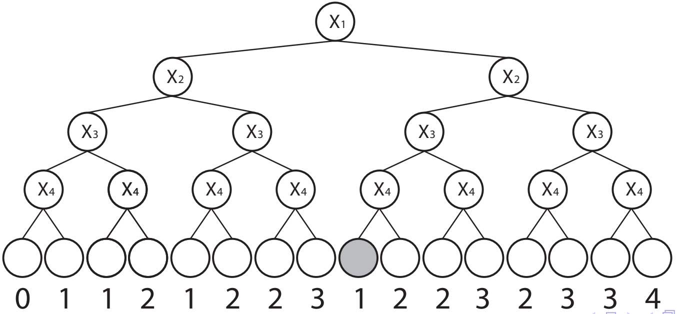
Depth-first Search (DFS) on an OR Tree

1. The search is guided by a **heuristic** $h(n)$
2. $h(n)$ is usually good, but **not perfect**
3. **Advantage:** memory efficient
4. **Drawback:** early mistakes are fatal



Limited Discrepancy Search (LDS) [Harvey and Ginsberg, 95]

1. **Discrepancy:** right turn (going against the heuristic)
2. **Leaf discrepancies:** number of right turns
3. There are $\binom{n}{k}$ leaves with k discrepancies
4. **LDS:** Search in increasing order of discrepancies
5. k-th iteration: visits leaves with k or less discrepancies



Algorithm

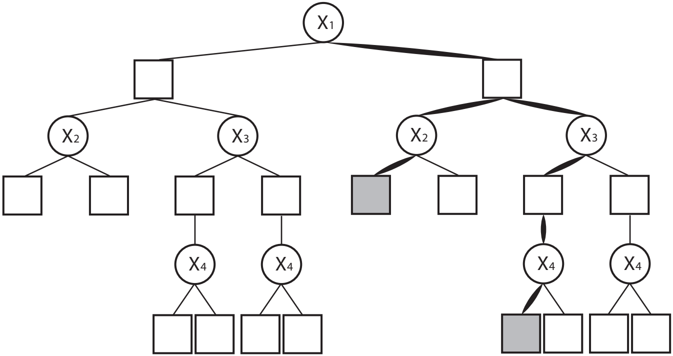
```
Function LDS()
begin
    for k = 0 ... n do
        if Probe(root, k) then return true
    return false
end
Function Probe(node, k)
begin
    if isLeaf(node) then return isGoal(node)
    if k = 0 then return Probe(left(node),0)
    else return (Probe(right(node),k − 1) or Probe(left(node),k))
end
```

Limited Discrepancy Search (LDS)

1. Successful in a number of domains
2. Several enhancements have been proposed (e.g. ILDS, DBDS,...)
3. In optimization problems (i.e, find best solution) LDS becomes an anytime algorithm

AND/OR search trees [Nilsson, 80]

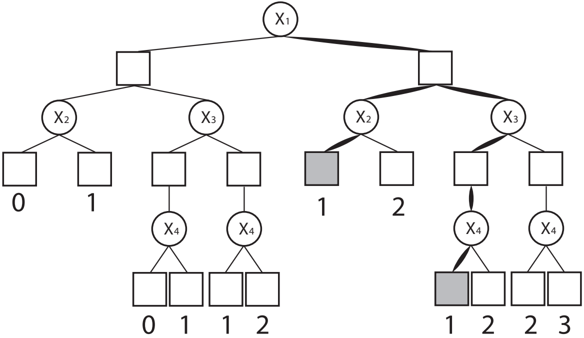
1. **OR nodes:** decision points
2. **AND nodes:** independent sub-problems
3. Solution tree
4. Depth-first AND/OR Search



Limited Discrepancy AND/OR search (LDSAO)

Definition

- Discrepancies of a leaf: right turns after OR nodes
- Discrepancies of a solution tree: **maximum** among branches



1. There are $O(n \cdot \binom{h}{k})$ solution trees with k discrepancies
2. **LDSAO:** searches solution trees in increasing number of discrepancies

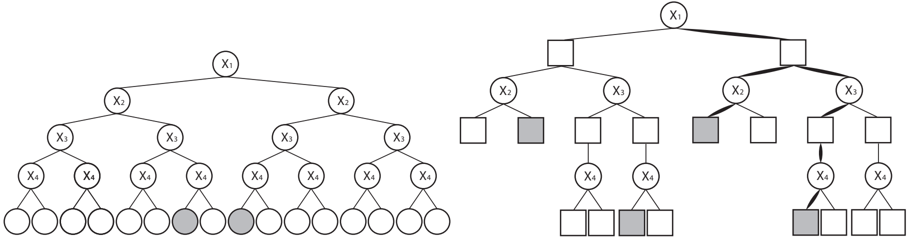
```
Function ProbeOr(nodeOr, k)
begin
    if k = 0 then return ProbeAnd(left(nodeOr),0)
    return ProbeAnd(right(nodeOr),k − 1) or ProbeAnd(left(nodeOr),k)
end
```

```
Function ProbeAnd(nodeAnd, k)
begin
    if isLeaf(nodeAnd) then
        return isGoal(nodeAnd)
    for nodeOr ∈ Successors(nodeAnd) do
        if not ProbeOr(nodeOr,k) then
            return false
    return true
end
```

Graphical Models

Search in Graphical Models can be represented with either OR trees or AND/OR trees (exploiting conditional independencies).

$$F(x_1, \ldots, x_4) = f(x_1, x_2) + f(x_1, x_3) + f(x_1, x_4) + f(x_3, x_4)$$



Graphical Models

Therefore, one can use LDS or LDSAO.

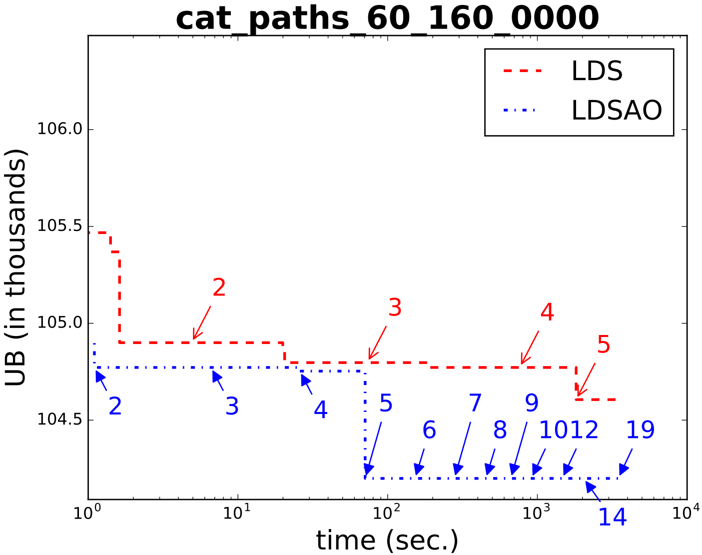
Properties

- LDSAO iterates faster than LDS (because paths are shorter in the AND/OR tree)
- LDSAO visits more complete assignments than LDS (because of k discrepancies in AND/OR may map to $> k$ discrepancies in OR)

Experimental Results

- Any-time performance of LDS vs LDSAO on the min-sum problem
- Heuristic: static MBE [Kask and Dechter 99; Ihler et al 2011] (i-bound set to 10, 15, 16)
- Six benchmarks (138 instances)
- Time limit: 1 hour

Experimental Results



Future Work

- Overcome the static ordering limitation of AND/OR search (dynamic variable orderings seem to be better)
- Overcome the non-any-time nature of LDSAO during each iteration
- Improvement of search effort (i.e, unbalanced AND/OR trees)
- Add AND/OR to LDS improvements