

ICS 6A
Solution to Homework Assignment 6
Winter 2004

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Answer the following questions (explain your answers).

1. Rosen, page 73, problem 2.

Using **TABLE 1** on page 58 of Rosen.

- a) Simplification
- b) Disjunctive syllogism
- c) Modus ponens
- d) Addition
- e) Hypothetical syllogism

2. Rosen, page 73, problem 5.

Answer: Universal instantiation is used to conclude that “If Socrates is a man, then Socrates is mortal.” Modus ponens is then used to conclude that Socrates is mortal.

3. Rosen, page 75, problem 13.

- a) NOT valid. Fallacy of affirming the conclusion.
- b) NOT valid. Fallacy of begging the question. (or fallacy of circular reasoning)
- c) Valid. Modus tollens.
- d) NOT valid. Fallacy of denying the hypothesis.

4. Rosen, page 75, problem 17.

See the book answer.

5. Rosen, page 75, problem 21.

See book’s answer.

6. Rosen, page 75, problem 24.

Prove that the product of two odd numbers is odd. Let $x = 2k + 1$ and $y = 2m + 1$ for some integer k and m . Their product $xy = (2k + 1)(2m + 1) = 4km + 2k + 2m + 1 = 2(2km + k + m) + 1$. Therefore xy is odd.

7. Rosen, page 75, problem 26.

Let r_1 and r_2 are two rational numbers, and $r_1 = \frac{a}{b}$, $r_2 = \frac{c}{d}$, where a, b, c, d are all integers. So:

$$r_1 \cdot r_2 = \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Since a, b, c, d are all integers, ac and bd are both integers. So $\frac{a \cdot c}{b \cdot d}$ is a rational number, which means that the product of two rational numbers r_1 and r_2 is rational.

8. Rosen, page 75, problem 29

See answer in book.

9. Rosen, page 223, problem 2.

Prove that if n is an odd positive integer, then $n^2 = 1 \pmod{8}$

Proof: If n is odd positive then $n = 2k + 1$. In that case $n^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1$. Since $k(k + 1)$ must be divisible by 2 we have that $4k(k + 1)$ is divisible by 8. Therefore when dividing n^2 by 8 we will have a remainder of 1.