ICS 6A
Solution to Homework Assignment 5

Winter 2004

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1. Rosen, page 392, problem 1.
   Answer: For a fair coin the probabilities of head and tail each time when it is flipped are: $P(head) = 0.5$ and $P(tail) = 0.5$ respectively. Assume $h = 1$ when the coin comes up with a head, and $h = 0$ when the coin comes up with a tail. So for each time when the coin is flipped $E(h) = 0.5 \times 1 + 0.5 \times 0 = 0.5$
   If the coin is flipped 10 times, $E(h) = 10 \cdot 0.5 = 5$

2. Rosen, page 392, problem 6.
   Answer: Let's assume:

   - The purchaser wins 10 million dollars as long as the ticket contains the six winning numbers chosen from the set $\{1, 2, 3, ..., 50\}$, no matter what order of the six numbers is.
   - The six winning numbers chosen from the set $\{1, 2, 3, ..., 50\}$ are different, which means each number can be chosen once.

   The probability that the purchaser wins 10 million dollars is: $P(win10million) = \frac{1}{C(50,6)} = \frac{1}{15890700}$.
   The probability that the purchaser wins nothing is: $P(win0) = 1 - P(win10million) = \frac{15890699}{15890700}$. So the expected amount of money is:
   $E(money) = (10 \times 10^6 - 1) \times P(win10million) + (0-1) \times P(win0) = 9999999 \times \frac{1}{15890700} + (-1) \times \frac{15890699}{15890700} = -0.37$
   which means the expected value the purchaser can "win" is -37 cents, where "-" means he/she loses 37 cents because he/she has to spend 1 dollar to buy the lottery ticket.

3. Rosen page 392, problem 7. (answer in the book)

4. Rosen, page 392, problem 8.
   Answer: Let x1 be the number that appears when a fair die #1 is rolled, x2 be the number that appears when a fair die #2 is rolled, x3 be the number that appears when a fair die #3 is rolled, X be the sum of the numbers that appear when three fair dice are rolled, then:
   $E(x1) = \sum_{i=1}^{i=6}(x1 \cdot P(x1 = i)) = \sum_{i=1}^{i=6}(x1 \cdot \frac{1}{6}) = \frac{1}{6} \cdot (1 + 2 + 3 + 4 + 5 + 6) = 3.5$
   Similarly, $E(x2) = E(x3) = 3.5$
   So $E(X) = E(x1 + x2 + x3) = E(x1) + E(x2) + E(x3) = 3.5 + 3.5 + 3.5 = 10.5$

5. Rosen page 392, problem 16.
   X and Y are not independent because if X=2 the probability of Y=2 is zero while not knowing X the probability of Y=2 is 1/4.

6. Rosen, page 393, problem 23.
   Answer: Please read **EXAMPLE 22** on page 281 of Rosen. When n independent Bernoulli trials are performed, where p is the probability of success on each trial, the variance of the number of successes V(x) = npq = np(1-p). Here n = 10, p = $\frac{1}{2}$, $V(x) = 10 \cdot \frac{1}{2} \cdot (1 - \frac{1}{2}) = 2.5$

7. Rosen, page 393, problem 24.
   Answer: Similar as problem 23, here n = 10, p = $\frac{1}{6}$, $V(x) = 10 \cdot \frac{1}{6} \cdot (1 - \frac{1}{6}) = \frac{25}{18}$

8. Rosen, page 394, problem 40.
The probability of each ball falling into the first bin is $\frac{1}{n}$. Since there are m balls each one represent a berouli trial with a probability for success $\frac{1}{n}$. The expected number of successes is $\frac{m}{n}$.

9. Suppose a discrete math class is given twice a week on Tuesday and Thursday. Let X be the random variable having the value "yes" if the attendance in the class is greater than 90 percent and "no" otherwise. Lets Y be the variable indicating the day in which the class is given. Suppose that

$$P(X = yes|Y = Thursday) = .9$$

and

$$P(X = yes|Y = Tuesday) = .6$$

Suppose you are told that in todays class there were more than 90 percent of the students. What is the probability that it is Tuesday? Namely, what is

$$P(Y = Tuesday|X = yes)?$$

Answer:

$$\begin{aligned}
P(Y = Tuesday|X = yes) &= \frac{P(Y=Tuesday \cap X=yes)}{P(X=yes)} \\
&= \frac{P(X=yes|Y=Tuesday) \cdot P(Y=Tuesday)}{P(X=yes)} \\
&= \frac{P(X=yes|Y=Tuesday) \cdot P(Y=Tuesday)}{P(X=yes \cap Y=Tuesday) + P(X=yes \cap Y=Thursday)} \\
&= \frac{P(X=yes|Y=Tuesday) \cdot P(Y=Tuesday)}{P(X=yes|Y=Tuesday) \cdot P(Y=Tuesday) + P(X=yes|Y=Thursday) \cdot P(Y=Thursday)} \\
&= \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.9 \cdot 0.5} \\
&= 0.4
\end{aligned}$$