ICS 6A
Solution to Homework Assignment 3
Winter 2004

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1. Rosen, page 319, problem 2.
   Proof: Because there are only 26 possible letters and there are 30 students, according to "THE PIGEONHOLE PRINCIPLE", at least 2 students have last names that begin with the same letter.

2. Rosen, page 320, problem 40.
   Proof: From 1000 to 1099 (inclusive) there are 100 numbers. We devide the 100 numbers into 50 boxes and each box contains two consecutive integers (1000 + 2k) and (1000 + 2k + 1), where k = 0, 1, ..., 49. For example box 0 contains 1000 and 1001, box 1 contains 1002 and 1003, ..., box 49 contains 1098 and 1099. Since there are 50 boxes and 51 houses, according to "THE PIGEONHOLE PRINCIPLE", at least 2 houses fall into the same box, which means at least two houses have addresses that are consecutive integers.

3. Rosen, page 324, problem 6.
   a) $C(5,1) = \frac{5!}{1! \cdot (5-1)!} = 5$
   b) $C(5,3) = \frac{5!}{3! \cdot (5-3)!} = 10$
   c) $C(8,4) = \frac{8!}{4! \cdot (8-4)!} = 70$
   d) $C(8,8) = \frac{8!}{8! \cdot (8-8)!} = 1$
   e) $C(8,0) = \frac{8!}{0! \cdot (8-0)!} = 1$
   f) $C(12,6) = \frac{12!}{6! \cdot (12-6)!} = 924$

4. Rosen, page 325, problem 8.
   Five runners can finish a race in 5! = 120 different orders if no ties are allowed.

5. Rosen, page 325, problem 20.
   Number of bit strings of length 10 have
   a) exactly three 0s: all other seven bits are 1s, so $C(10,3) = 120$
   b) more 0s than 1s: six 0s and four 1s or seven 0s and three 1s, or 8 0s and two 1s or nine 0s and one 1s or all 0s is: $C(10,6) + C(10,7) + C(10,8) + C(10,9) + C(10,10)$
   c) at least seven 1s: at most three 0s

   - number of bit strings of length 10 have exact three 0s: $C(10,3) = 120$ (from result of a)
   - number of bit strings of length 10 have exact two 0s: $C(10,2) = 45$
   - number of bit strings of length 10 have exact one 0s: $C(10,1) = 10$
   - number of bit strings of length 10 have no 0: all ten 1s, 1

   So number of bit strings of length 10 have at least seven 1s: $120 + 45 + 10 + 1 = 176$

   d) at least three 1s:

   - Total number of bit strings of length 10: $2^{10} = 1024$
   - number of bit strings of length 10 have exact two 1s: $C(10,2) = 45$
   - number of bit strings of length 10 have exact one 1s: $C(10,1) = 10$
   - number of bit strings of length 10 have no 1: all ten 0s, 1

So number of bit strings of length 10 have at least three 1s: $1024 - (45 + 10 + 1) = 968$

6. Rosen, page 326, problem 27.
   a) Choose four members from 25 members: $C(25, 4) = 12650$
   b) Choose a president, vice president, secretary, and treasurer: $P(25, 4) = 303600$

7. Rosen, page 326, problem 31.
   21 consonants and 5 vowels in English. Number of strings of six lowercase letters of the English alphabet contain
   a) exactly 1 vowel: 1 vowel and 5 consonants
   number of possible position for the vowel: $C(6, 1) = 6$
   so $C(6, 1) * 5^1 * 21^5 = 122523030$
   b) exactly 2 vowels: 2 vowels and 4 consonants
   number of possible position for the vowel: $C(6, 2) = 15$
   so $C(6, 2) * 5^2 * 21^4 = 72930375$
   c) at least 1 vowel:
   Total number of strings of six lowercase letters: $26^6 = 308915776$
   number of strings of six lowercase letters contain NO vowel: $21^6 = 85766121$
   so number of strings of six lowercase letters contain at least 1 vowel: $26^6 - 21^6 = 223149655$
   d) at least 2 vowel:
   Total number of strings of six lowercase letters: $26^6 = 308915776$
   number of strings of six lowercase letters contain NO vowel: $21^6 = 85766121$
   number of strings of six lowercase letters contain exactly 1 vowel: 122523030 (result of b)
   so number of strings of six lowercase letters contain at least 2 vowel: $26^6 - 21^6 - 122523030 = 100626625$

8. Rosen, page 342, problem 5.
   There are $5^3 = 125$ ways to assign three jobs to five employees if each employee can be given more than one job.

9. Rosen, page 342, problem 9 a,c,d.
   There are totally 8 kinds of bagels. Number of ways to choose
   a) six bagels: $C(8 + 6 - 1, 6) = 1716$
   c) two dozen bagels: $C(8 + 24 - 1, 24) = 2629575$
   d) a dozen bagels with at least one of each kind:
   take out 1 bagel from each kind. Only $12 - 8 = 4$ bagels need to be chosen.
   so there are $C(8 + 4 - 1, 4) = 330$ ways to choose a dozen bagels with at least one of each kind.

10. Rosen, page 343, problem 32. Using all letters in "*AARDVARK*"
    Answer: There are 5 letters except 3 "*A*"s. Because 3 "*A*"s must be consecutive, we can take 3 "*A*"s as 1 letter. So there are totally 6 letters. The permutation of 6 letters are 6!. But there 2 identical letters "*R*", so the total number of strings can be made is $6!/2 = 360$.

11. Rosen, page 343, problem 36. six 1s and eight 0s
    Answer: It is equivalent with "Choose six positions for six 1s in the string.", so the number of bit strings can be formed is $C(6 + 8, 6) = 3003$