Backtracking search: look-back

ICS 275

Spring 2010

Look-back:
Backjumping / Learning

- Backjumping:
  - In deadends, go back to the most recent culprit.
- Learning:
  - constraint-recording, no-good recording.
  - good-recording

Backjumping

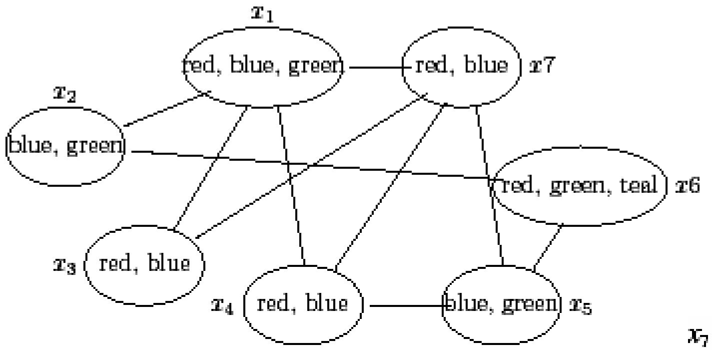
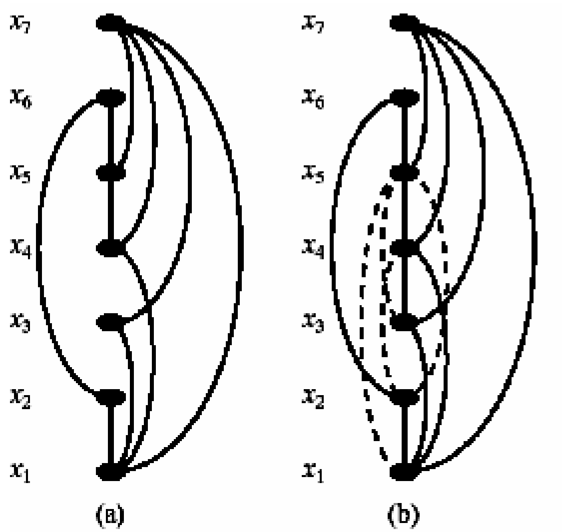


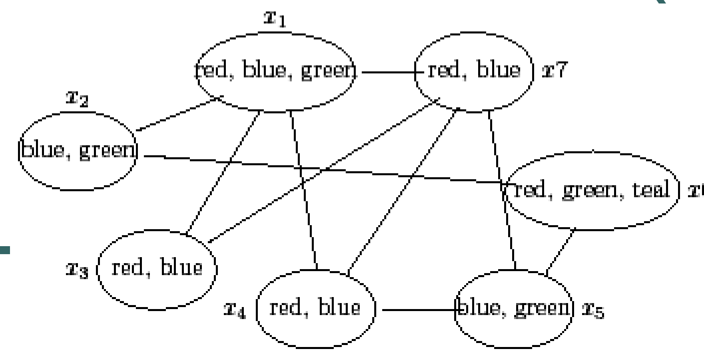
Figure 6.1: A modified coloring problem.

- (X1=r,x2=b,x3=b,x4=b,x5=g,x6=r,x7={r,b})
- (r,b,b,b,g,r) **conflict set** of x7
- (r,-,b,b,g,-) c.s. of x7
- (r,-,b,-,-,-,-) **minimal conflict-set**
- **Leaf deadend**: (r,b,b,b,g,r)
- Every conflict-set is a **no-good**



Gaschnig jumps only at leaf-dead-ends

Internal dead-ends: dead-ends that are non-leaf



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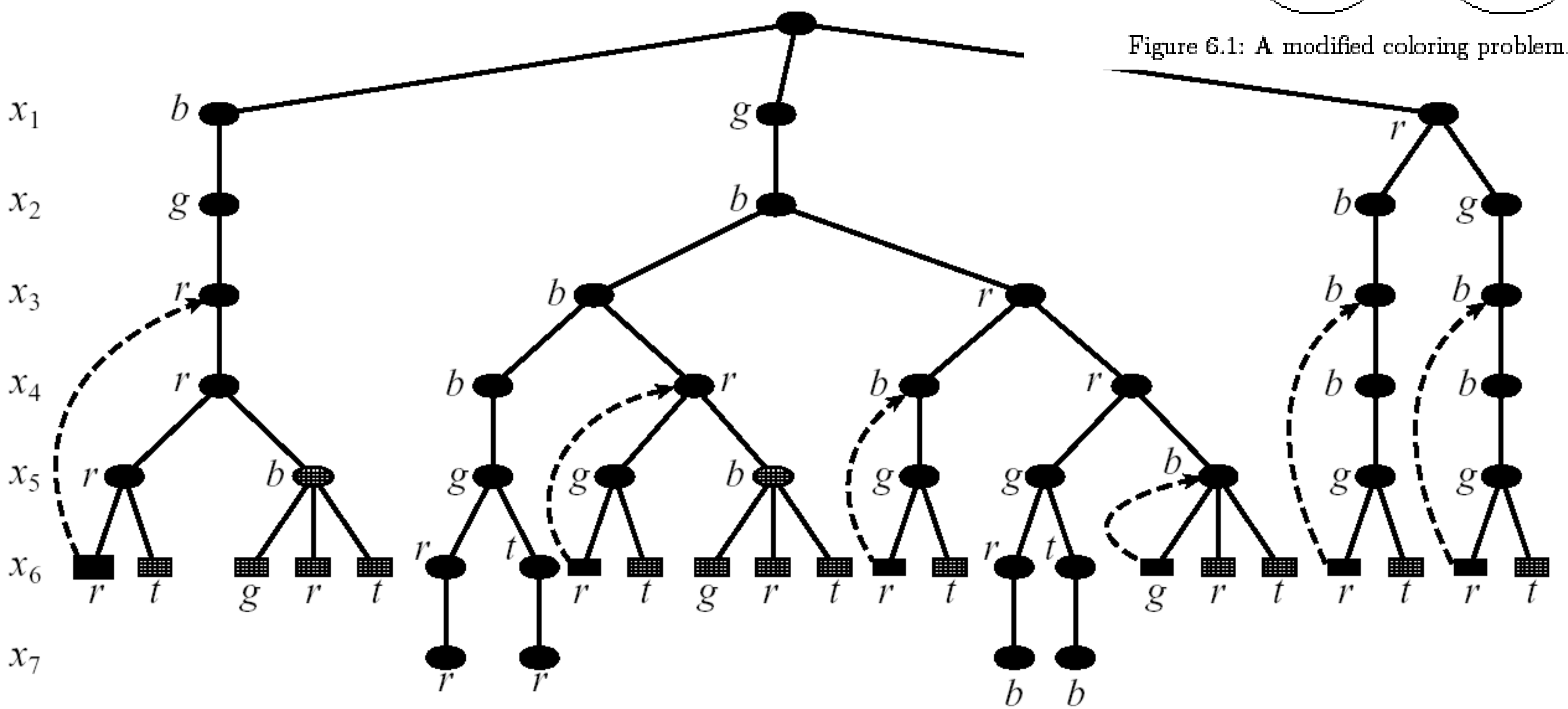

Example 6.3.1 In Figure 6.4, all of the backjumps illustrated lead to internal dead-ends, except for the jump back to $(\langle x_1, green\rangle, \langle x_2, blue\rangle, \langle x_3, red\rangle, \langle x_4, blue\rangle)$, because this is the only case where another value exists in the domain of the culprit variable. □

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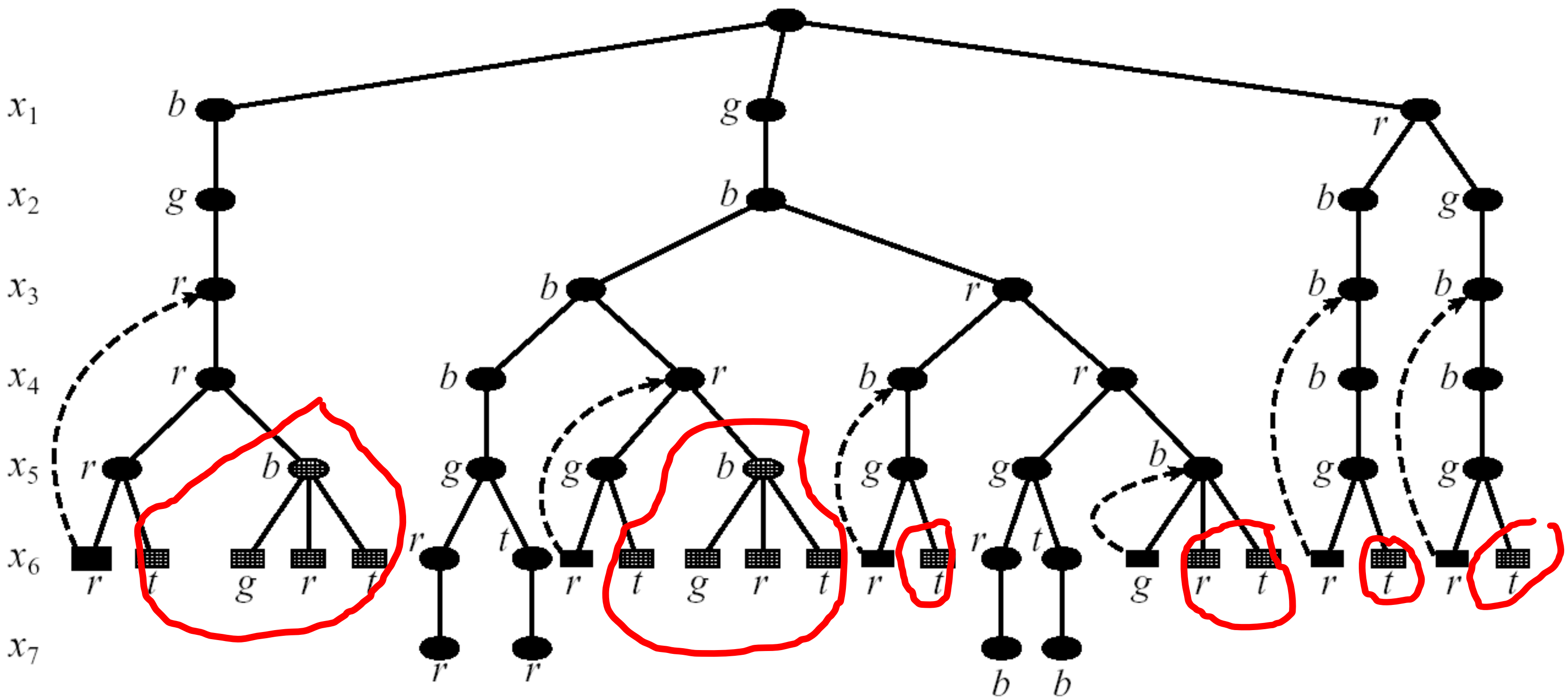

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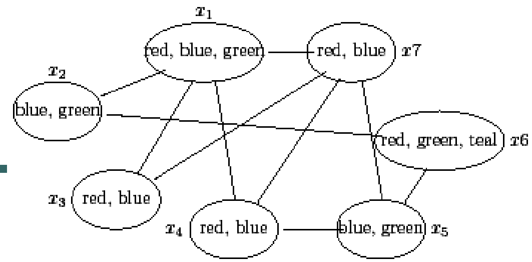
Backjumping styles

- Jump at leaf only (Gaschnig 1977)
  - Context-based
- Graph-based (Dechter, 1990)
  - Jumps at leaf and internal dead-ends, graph information
- Conflict-directed (Prosser 1993)
  - Context-based, jumps at leaf and internal dead-ends

Gaschnig's backjumping:

Culprit variable



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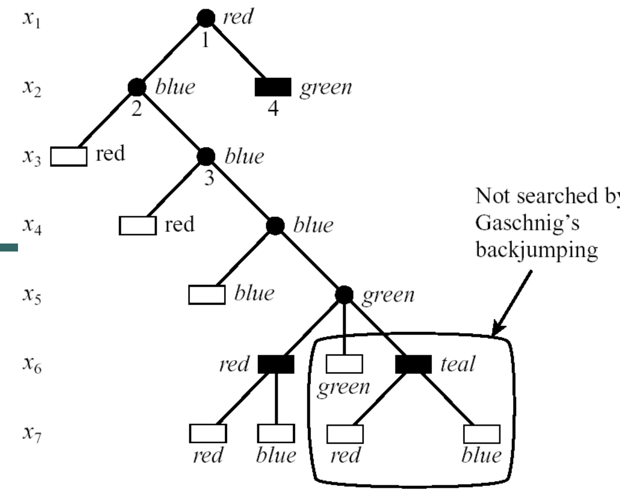


Definition 6.2.1 (culprit variable) *Let $\vec{a_i} = (a_1, ..., a_i)$ be a leaf dead-end. The* culprit index *relative to $\vec{a_i}$ is defined by $b = \min\{j \leq i |\ \vec{a_j}$ conflicts with $x_{i+1}\}$. We define the* culprit variable *of $\vec{a_i}$ to be x_b.*

- *If a_i is a leaf deadend and x_b its culprit variable, then a_b is a safe backjump destination and a_j, j<b is not.*
- *The culprit of x7 (r,b,b,b,g,r) is (r,b,b) → x3*

Gaschnig's backjumping Implementation [1979]

- Gaschnig uses a marking technique to compute culprit.
- Each variable xj maintains a pointer (latest_j) to the latest ancestor incompatible with any of its values.
- While forward generating $\vec{a}_i$, keep array latest_i, 1<=j<=n, of pointers to the last value conflicted with some value of x_j
- The algorithm jumps from a leaf-dead-end x_{i+1} back to latest_(i+1) which is its culprit.

Gaschnig's backjumping

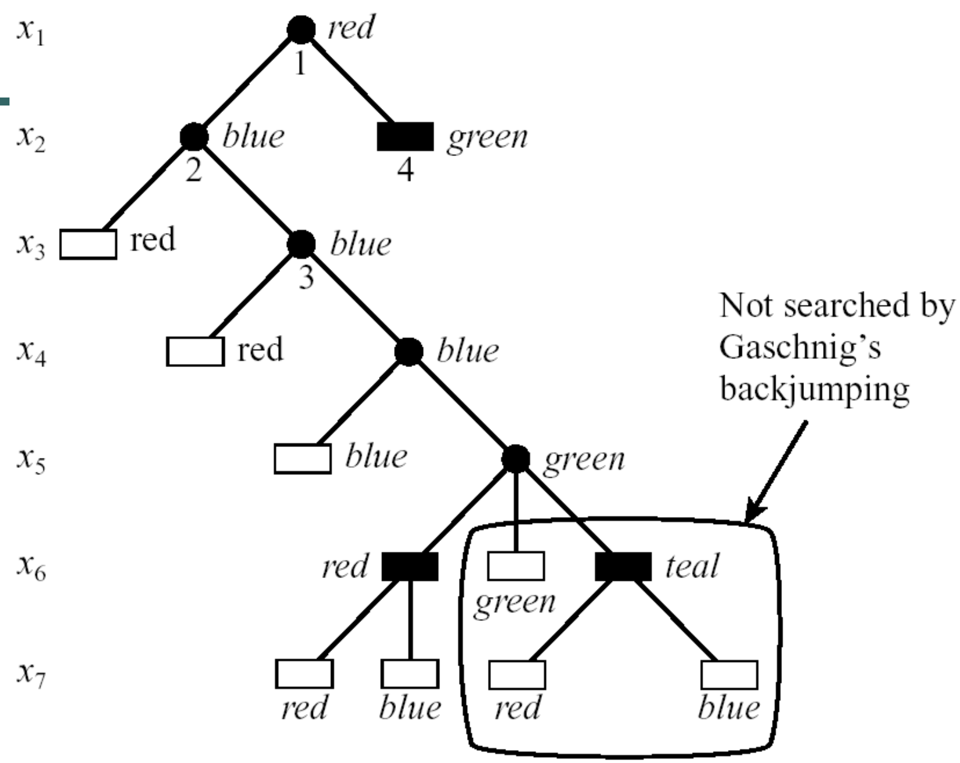
procedure GASCHNIG'S-BACKJUMPING
Input: A constraint network $\mathcal{R} = (X, D, C)$
Output: Either a solution, or a decision that the network is inconsistent.

    $i \leftarrow 1$ (initialize variable counter)
    $D'_i \leftarrow D_i$ (copy domain)
    $latest_i \leftarrow 0$ (initialize pointer to culprit)
    while $1 \leq i \leq n$
        instantiate $x_i \leftarrow$ SELECTVALUE-GBJ
        if x_i is null (no value was returned)
            $i \leftarrow latest_i$ (backjump)
        else
            $i \leftarrow i + 1$
            $D'_i \leftarrow D_i$
            $latest_i \leftarrow 0$
    end while
    if $i = 0$
        return "inconsistent"
    else
        return instantiated values of $\{x_1, \ldots, x_n\}$
end procedure

procedure SELECTVALUE-GBJ
    while D'_i is not empty
        select an arbitrary element $a \in D'_i$, and remove a from D'_i
        $consistent \leftarrow true$
        $k \leftarrow 1$
        while $k < i$ and *consistent*
            if $k > latest_i$
                $latest_i \leftarrow k$
            if not CONSISTENT$(\vec{a_k}, x_i = a)$
                $consistent \leftarrow false$
            else
                $k \leftarrow k + 1$
        end while
        if *consistent*
            return a
    end while
    return null (no consistent value)
end procedure

Figure 6.3: Gaschnig's backjumping algorithm.

Example of Gaschnig's backjump



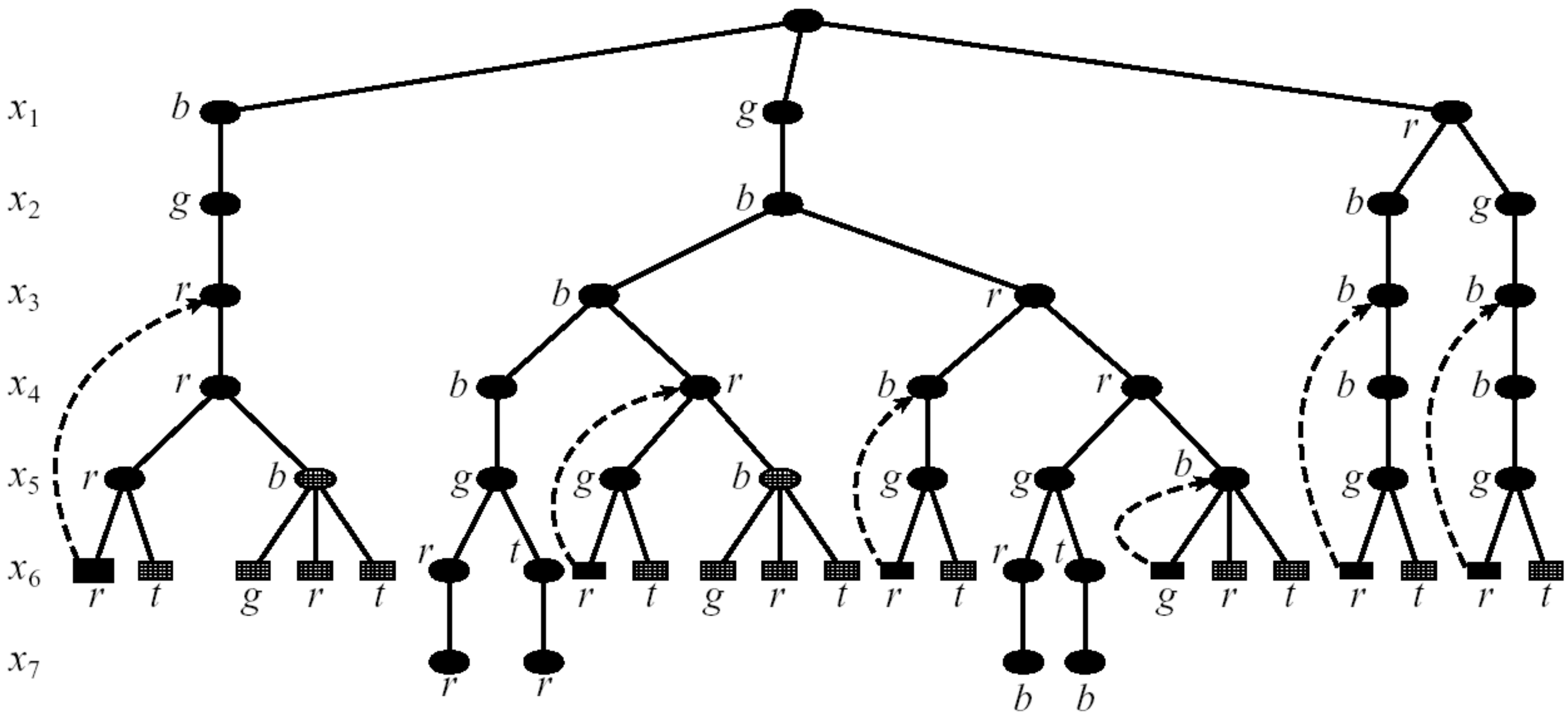
Example 6.2.3 Consider the problem in Figure 6.1 and the order d_1. At the dead-end for x_7 that results from the partial instantiation $(< x_1, red >, < x_2, blue >, < x_3, blue >, < x_4, blue >, < x_5, green >, < x_6, red >)$, $latest_7 = 3$, because $x_7 = red$ was ruled out by $< x_1, red >$, $x_7 = blue$ was ruled out by $< x_3, blue >$, and no later variable had to be examined. On returning to x_3, the algorithm finds no further values to try ($D'_3 = \emptyset$). Since $latest_3 = 2$, the next variable examined will be x_2. Thus we see the algorithm's ability to backjump at leaf dead-ends. On subsequent dead-ends, as in x_3, it goes back to its preceding variable only. An example of the algorithm's practice of pruning the search space is given in Figure 6.2. □

Properties

- Gaschnig's backjumping implements only safe and maximal backjumps in leaf-deadends.

Gaschnig jumps only at leaf-dead-ends

Internal dead-ends: dead-ends that are non-leaf



Example 6.3.1 In Figure 6.4, all of the backjumps illustrated lead to internal dead-ends, except for the jump back to $(\langle x_1, green\rangle, \langle x_2, blue\rangle, \langle x_3, red\rangle, \langle x_4, blue\rangle)$, because this is the only case where another value exists in the domain of the culprit variable. □

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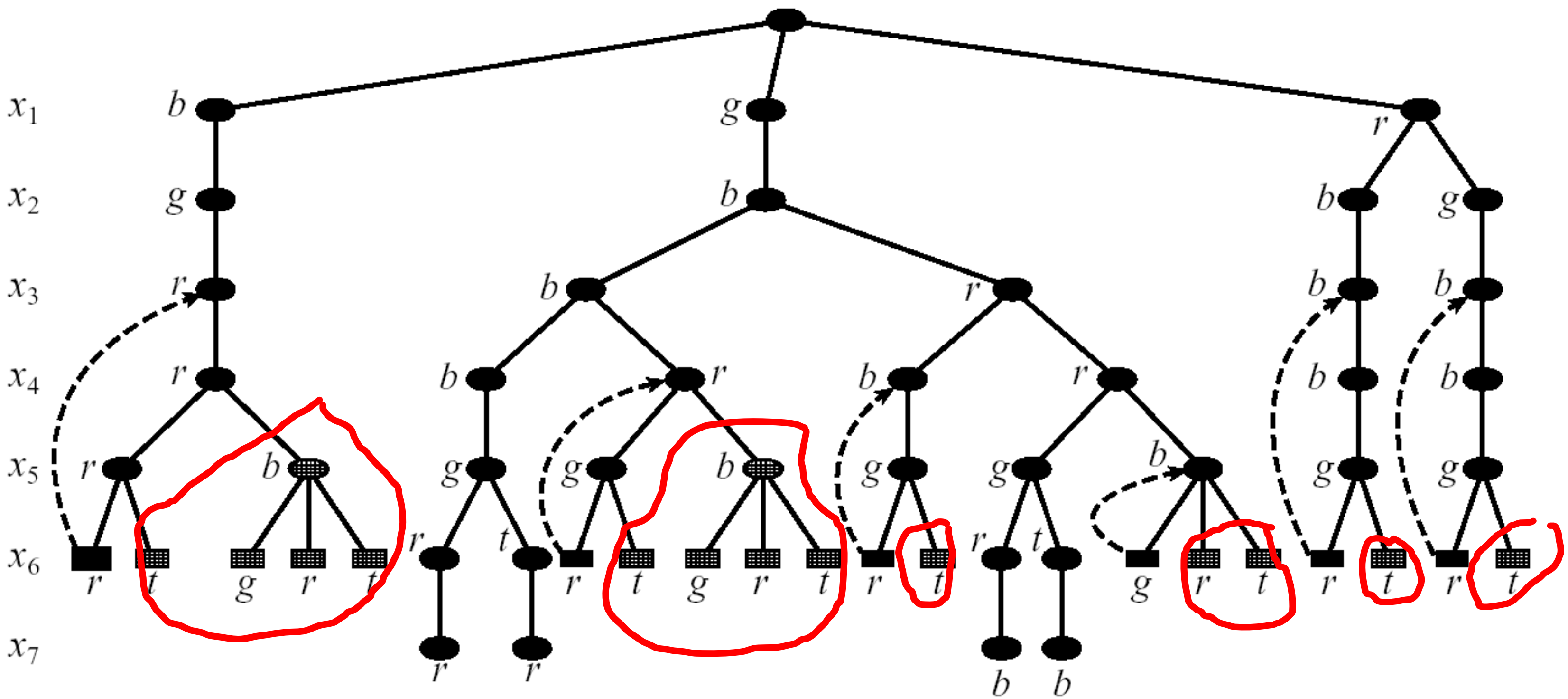

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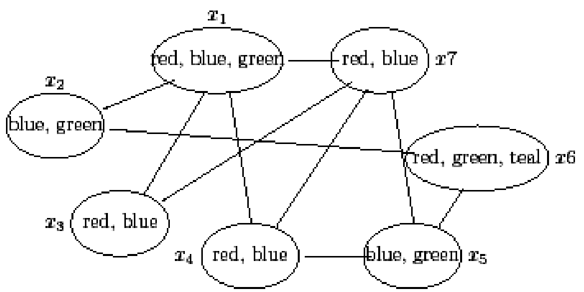
Graph-based backjumping scenarios Internal deadend at X4

- Scenario 1, deadend at x4:
- Scenario 2: deadend at x5:
- Scenario 3: deadend at x7:
- Scenario 4: deadend at x6:



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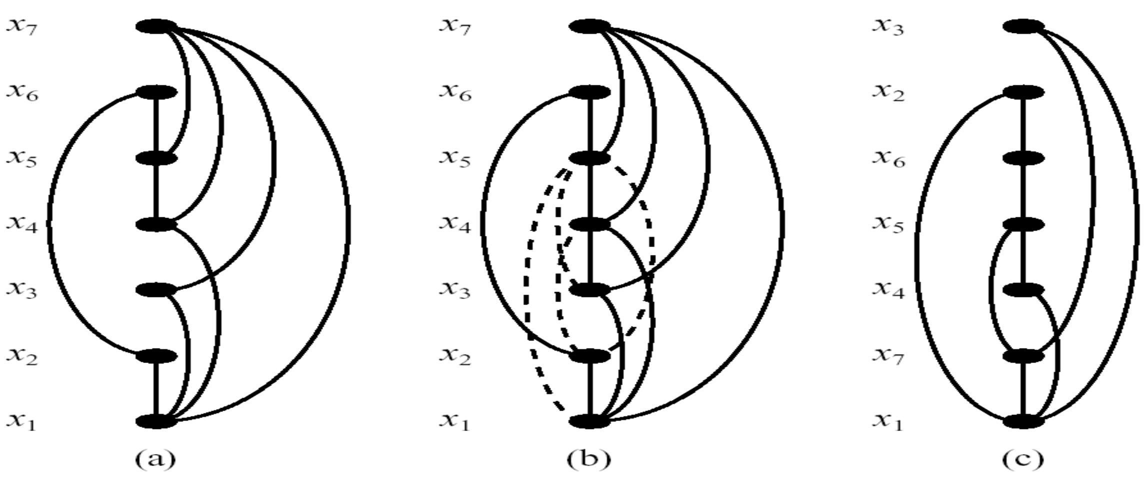
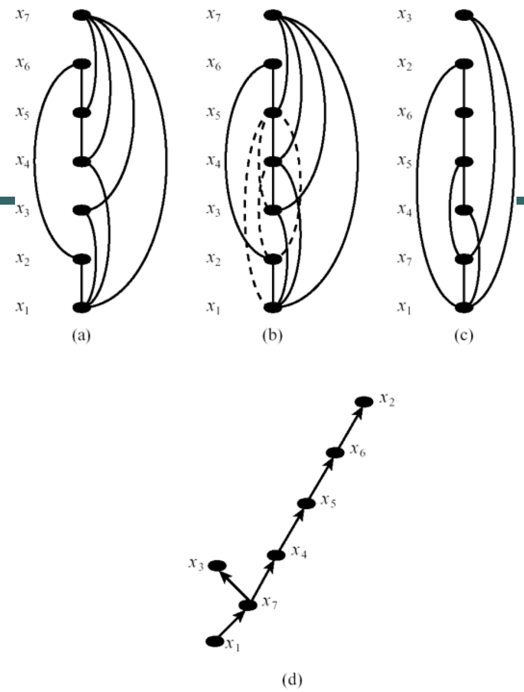


Graph-based backjumping

- Uses only graph information to find culprit
- Jumps both at leaf and at internal dead-ends
- Whenever a deadend occurs at x, it jumps to the most recent variable y connected to x in the graph. If y is an internal deadend it jumps back further to the most recent variable connected to x or y.
- The analysis of conflict is approximated by the graph.
- Graph-based algorithm provide graph-theoretic bounds.

Ancestors and parents

- anc(x7) = {x5,x3,x4,x1}
- p(x7) =x5
- p(r,b,b,b,g,r) = x5



Definition 6.3.2 (ancestors, parent) *Given a constraint graph and an ordering of the nodes* $\mathbf{d}$*, the* ancestor set *of variable* x*, denoted* $anc(x)$*, is the subset of the variables that precede and are connected to* x*. The* parent *of* x*, denoted* $p(x)$*, is the most recent (or latest) variable in* $anc(x)$*. If* $\vec{a_i} = (a_1, ..., a_i)$ *is a leaf dead-end, we equate* $anc(\vec{a_i})$ *with* $anc(x_{i+1})$*, and* $p(\vec{a_i})$ *with* $p(x_{i+1})$.

Internal deadends analysis

Definition 6.3.5 (session) *We say that backtracking* invisits x_i *if it processes* x_i *coming from a variable earlier in the ordering. The session of* x_i *starts upon the invisiting of* x_i *and ends when retracting to a variable that precedes* x_i*. At a given state of the search where variable* x_i *is already instantiated, the* current session *of* x_i *is the set of variables processed by the algorithm since the most recent* invisit *to* x_i*. The current session of* x_i *includes* x_i *and therefore the session of a leaf dead-end variable has a single variable.*

Definition 6.3.6 (relevant dead-ends) *The relevant dead-ends of* x_i*'s session are defined recursively as follows. The relevant dead-ends of a leaf dead-end* x_i*, denoted* $r(x_i)$*, is* x_i*. If* x_i *is variable to which the algorithm retracted from* x_j*, then the relevant-dead-ends of* x_i *are the union of its current relevant dead-ends and the ones inherited from* x_j*, namely,* $r(x_i) = r(x_i) \cup r(x_j)$.

Definition 6.3.7 (induced ancestors, induced parent) *Let* x_i *be a variable that is an internal or leaf dead-end. Let* Y *be a subset of the variables consisting of all its relevant dead-ends in the current session of* x_i*. We denote* $anc(Y) = \cup_{y \in Y} anc(y)$*. The* induced ancestor set *of* x_i *relative to* Y*,* $I_i(Y)$*, is the union of all* Y*'s ancestors, restricted to variables that precede* x_i*. Formally,* $I_i(Y) = anc(Y) \cap \{x_1, ..., x_{i-1}\}$*. The induced parent of* x_i *relative to* Y*,* $P_i(Y)$*, is the latest variable in* $I_i(Y)$*. We call* $P_i(Y)$ *the graph-based culpribt of* x_i*.*

Graph-based backjumping algorithm, but we need to jump at internal deadends too

procedure GRAPH-BASED-BACKJUMPING
Input: A constraint network $\mathcal{R} = (X, D, C)$
Output: Either a solution, or a decision that the network is inconsistent.

```
    compute anc(x_i) for each x_i     (see Definition 6.3.2 in text)
    i ← 1                             (initialize variable counter)
    D'_i ← D_i                        (copy domain)
    I_i ← anc(x_i)                    (copy of anc() that can change)
    while 1 ≤ i ≤ n
       instantiate x_i ← SELECTVALUE
       if x_i is null                 (no value was returned)
          iprev ← i
          i ←latest index in I_i      (backjump)
          I_i ← I_i ∪ I_iprev − {x_i}
       else
          i ← i + 1
          D'_i ← D_i
          I_i ← anc(x_i)
    end while
    if i = 0
       return "inconsistent"
    else
       return instantiated values of {x_1, . . . , x_n}
end procedure

procedure SELECTVALUE                 (same as BACKTRACKING's)
    while D'_i is not empty
       select an arbitrary element a ∈ D'_i, and remove a from D'_i
       if CONSISTENT(a⃗_{i−1}, x_i = a)
          return a
    end while
    return null                       (no consistent value)
end procedure
```

Figure 6.5: The graph-based backjumping algorithm.

When not all variables
In the session above
X_i are relevant deadends?
See example 6.6

Properties of graph-based backjumping

- Algorithm graph-based backjumping jumps back at any deadend variable as far as graph-based information allows.
- For each variable, the algorithm maintains the induced-ancestor set I_i relative the relevant dead-ends in its current session.
- The size of the induced ancestor set is at most $w^*(d)$.

Conflict-directed backjumping
(Prosser 1990)

- Extend Gaschnig's backjump to internal dead-ends.
- Exploits information gathered during search.
- For each variable the algorithm maintains an induced **jumpback set**, and jumps to most recent one.
- **Use the following concepts:**
  - An ordering over variables induced a strict ordering between constraints**:** $R_1<R_2<\ldots R_t$
  - Use **earliest minimal consflict-set (**emc(x_(i+1)) **)** of a deadend.
  - Define the **jumpback set** of a deadend

Example of conflict-directed backjumping

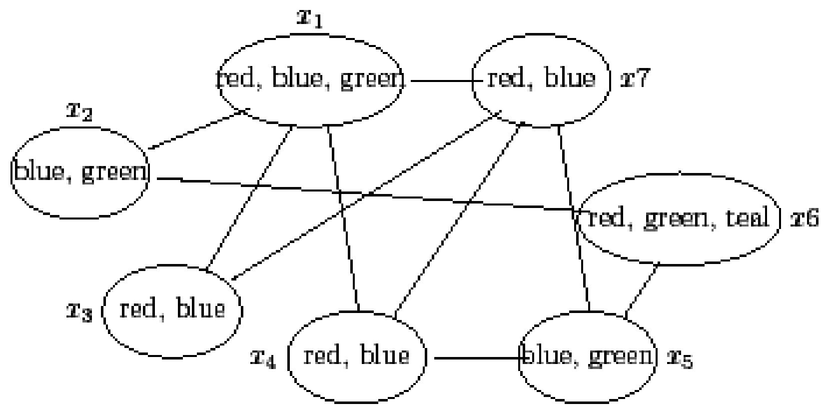


Figure 6.1: A modified coloring problem.

Example 6.4.5 Consider the problem of Figure 6.1 using ordering $d_1 = (x_1, \ldots, x_7)$. Given the dead-end at x_7 and the assignment $\vec{a_6} = (blue, green, red, red, blue, red)$, the emc set is $(< x_1, blue >, < x_3, red >)$, since it accounts for eliminating all the values of x_7. Therefore, algorithm conflict-directed backjumping jumps to x_3. Since x_3 is an internal dead-end whose own $var - emc$ set is $\{x_1\}$, the jumpback set of x_3 includes just x_1, and the algorithm jumps again, this time back to x_1. □

Properties

- Given a dead-end $\vec{a}_i$, the latest variable in its jumpback set J_i is the earliest variable to which it is safe to jump.
- This is the culprit.
- Algorithm conflict-directed backtracking jumps back to the latest variable in the dead-ends's jumpback set, and is therefore safe and maximal.

Conflict-directed backjumping

procedure CONFLICT-DIRECTED-BACKJUMPING
Input: A constraint network $\mathcal{R} = (X, D, C)$.
Output: Either a solution, or a decision that the network is inconsistent.

```
    i ← 1                                  (initialize variable counter)
    D'_i ← D_i                             (copy domain)
    J_i ← ∅                                (initialize conflict set)
    while 1 ≤ i ≤ n
        instantiate x_i ← SELECTVALUE-CBJ
        if x_i is null                     (no value was returned)
            iprev ← i
            i ← index of last variable in J_i      (backjump)
            J_i ← J_i ∪ J_iprev − {x_i}(merge conflict sets)
        else
            i ← i + 1                      (step forward)
            D'_i ← D_i                     (reset mutable domain)
            J_i ← ∅                        (reset conflict set)
    end while
    if i = 0
        return "inconsistent"
    else
        return instantiated values of {x_1, . . . , x_n}
end procedure

subprocedure SELECTVALUE-CBJ

    while D'_i is not empty
        select an arbitrary element a ∈ D'_i, and remove a from D'_i
        consistent ← true
        k ← 1
        while k < i and consistent
            if CONSISTENT(a⃗_k, x_i = a)
                k ← k + 1
            else
                let R_S be the earliest constraint causing the conflict
                add the variables in R_S's scope S, but not x_i, to J_i
                consistent ← false
        end while
        if consistent
            return a
    end while
    return null                            (no consistent value)
end procedure
```


Figure 6.7: The conflict-directed backjumping algorithm.

Graph-based backjumping on DFS orderings

Example 6.5.1 Consider, once again, the CSP in Figure 6.1. A *DFS* ordering $d_2 = (x_1, x_7, x_4, x_5, x_6, x_2, x_3)$ and its corresponding *DFS* spanning tree are given in Figure 6.6c,d. If a dead-end occurs at node x_3, the algorithm retreats to its *DFS* parent, which is x_7. □

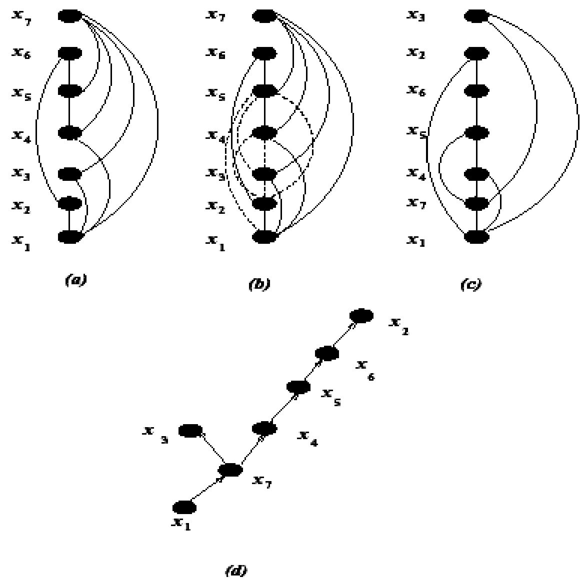


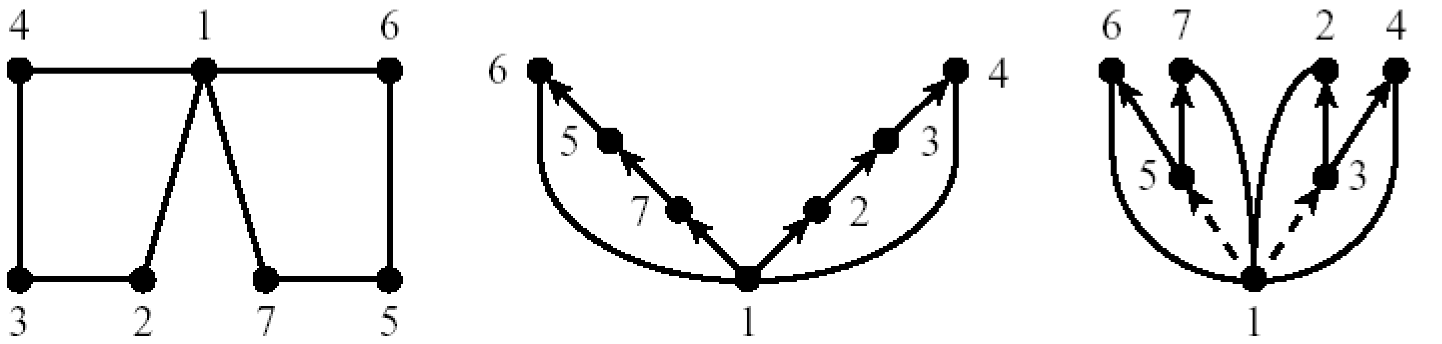
Figure 6.6: Several ordered constraint graphs of the problem in Figure 6.1: (a) along ordering $d_1 = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$, (b) the induced graph along d_1, (c) along ordering $d_2 = (x_1, x_7, x_4, x_5, x_6, x_2, x_3)$, and (d) a DFS spanning tree along ordering d_2.

Complexity of Graph-based Backjumping

- T_i= number of nodes in the AND/OR search space rooted at x_i (level m-i)
- Each assignment of a value to x_i generates subproblems:
  - T_i = k b T_{i-1}
  - T_0 = k
- Solution: $T_m = b^m k^{m+1}$

Theorem 6.5.3 *When graph-based backjumping is performed on a* DFS *ordering of the constraint graph, the number of nodes visited is bounded by* $O((b^m k^{m+1}))$, *where* b *bounds the branching degree of the* DFS *tree associated with that ordering,* m *is its depth and* k *is the domain size. The time complexity (measured by the number of consistency checks) is* $O(ek(bk)^m)$, *where* e *is the number of constraints.*

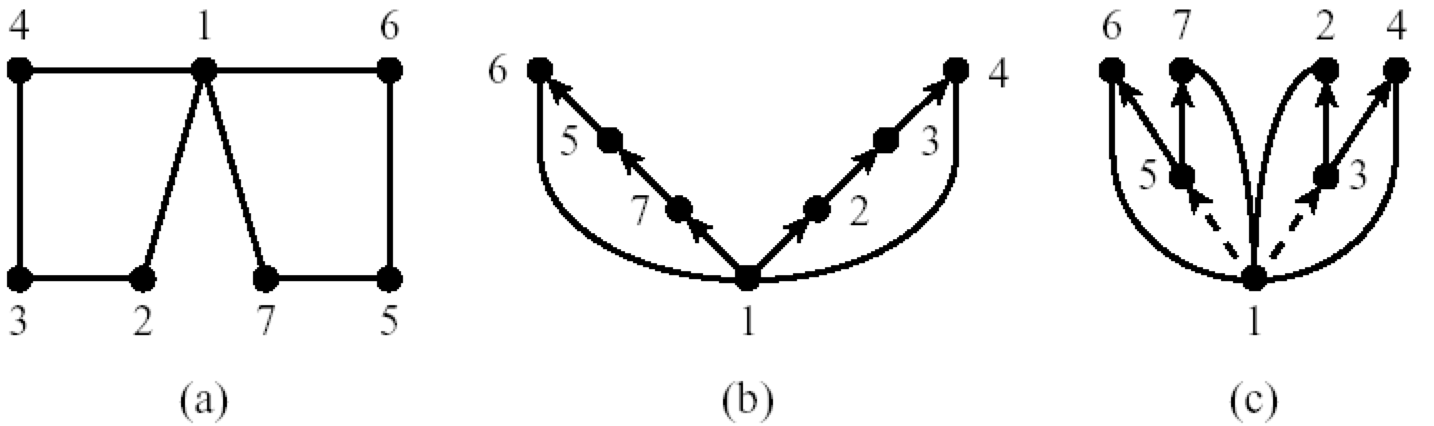
DFS of graph and induced graphs



(a) (b) (c)

Spanning-tree of a graph;
DFS spanning trees, BFS spanning trees.

Complexity of Backjumping uses pseudo-tree analysis



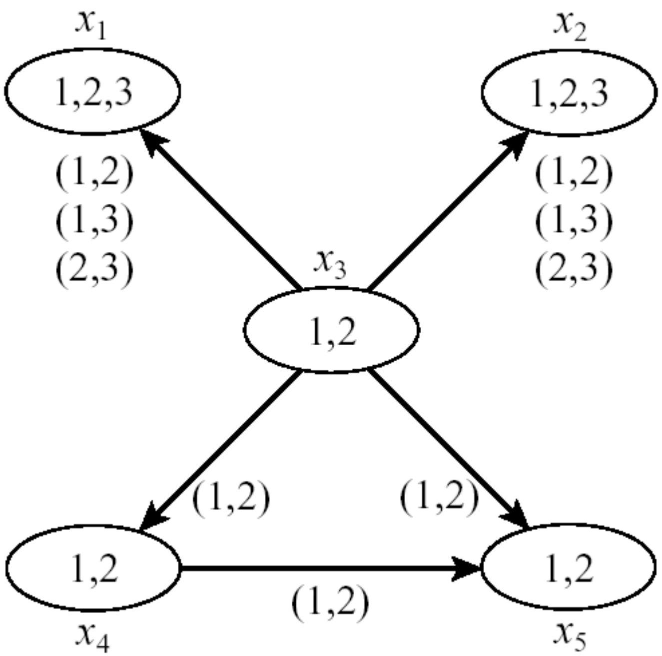
Simple: always jump back to parent in pseudo tree

Complexity for csp: exp(tree-depth)

Complexity for csp: exp(w*log n)

Look-back: No-good Learning

Learning means recording conflict sets used as constraints to prune future search space.



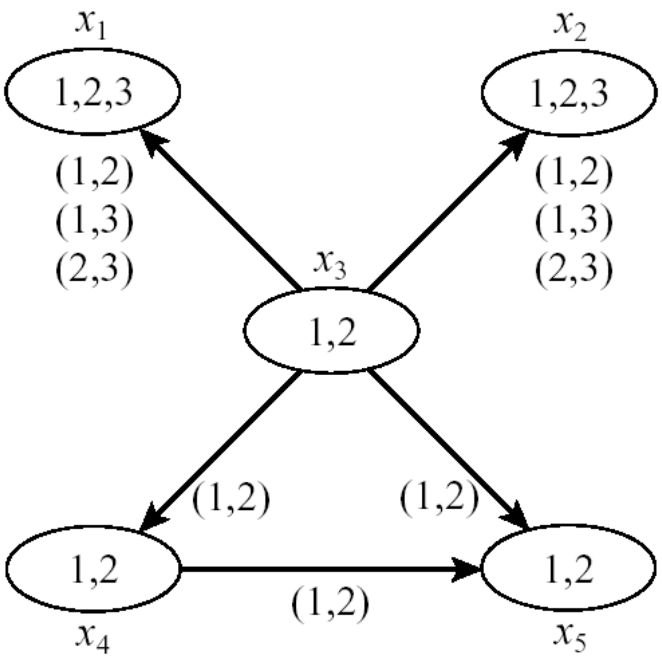
- (x1=2,x2=2,x3=1,x4=2) is a dead-end
- Conflicts to record:
  - (x1=2,x2=2,x3=1,x4=2) 4-ary
  - (x3=1,x4=2) binary
  - (x4=2) unary

Learning, constraint recording

- Learning means recording conflict sets
- An opportunity to learn is when deadend is discovered.
- Goal of learning to not discover the same deadends.
- Try to identify small conflict sets
- Learning prunes the search space.

Nogoods explain deadends

Learning means recording explanations to conflicts
They are implied constraints



- Conflicts to record are explanations
  - (x1=2,x2=2,x3=1,x4=2) 4-ary
  - (x1=2,x2=2,x3=1,x4=2)→ (x ≠1) and x_1=2
  - (x3=1,x4=2) → (x ≠1)
  - (x4=2) → (x ≠1)

Learning example

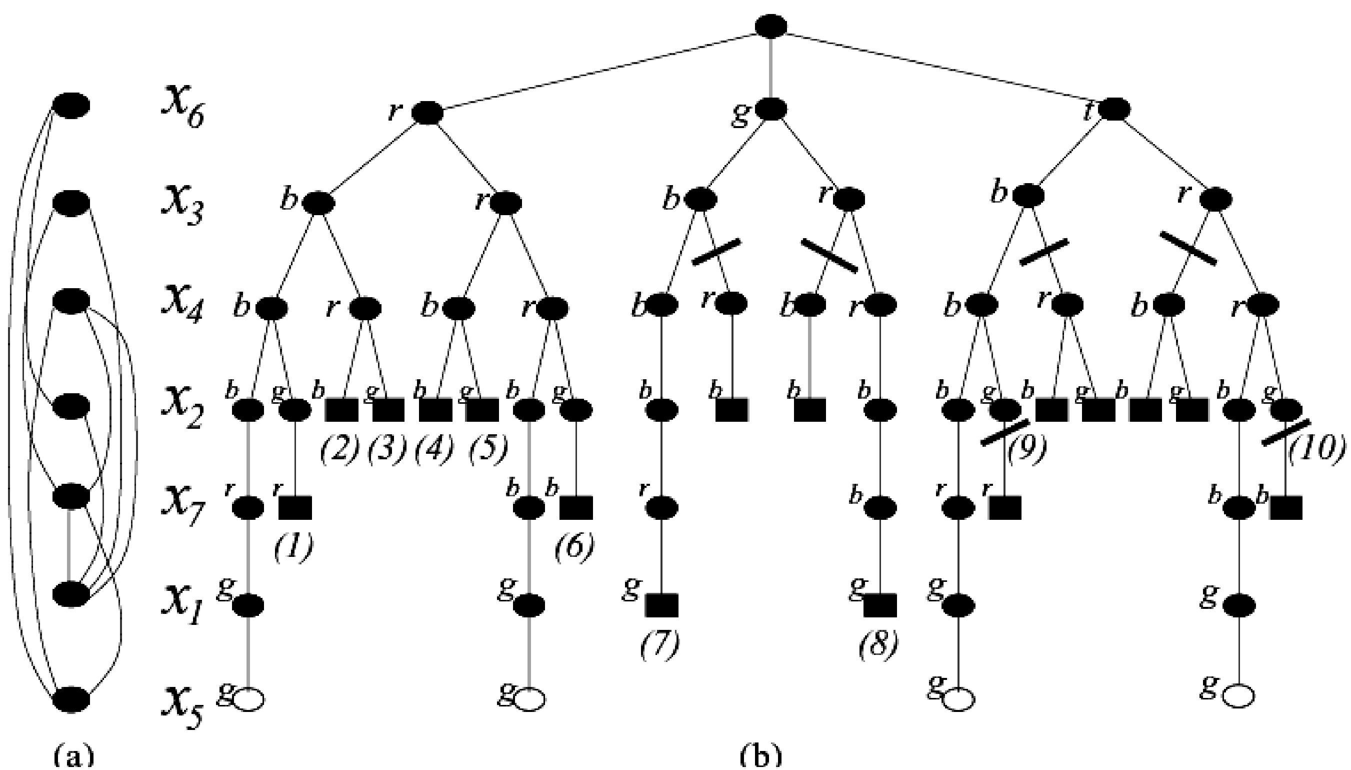


Figure 6.9: The search space explicated by backtracking on the CSP from Figure 6.1, using the variable ordering $(x_6, x_3, x_4, x_2, x_7, x_1, x_5)$ and the value ordering (*blue*, *red*, *green*, *teal*). Part (a) shows the ordered constraint graph, part (b) illustrates the search space. The cut lines in (b) indicate branches not explored when graph-based learning is used.

Learning Issues

- Learning styles
  - Graph-based or context-based
  - i-bounded, scope-bounded
  - Relevance-based
- Non-systematic randomized learning
- Implies time and space overhead
- Applicable to SAT

Graph-based learning algorithm

procedure GRAPH-BASED-BACKJUMP-LEARNING

instantiate $x_i \leftarrow$ SELECTVALUE
if x_i is null (no value was returned)
record a constraint prohibiting $\vec{a}_{i-1}[I_i]$.
$iprev \leftarrow i$
(algorithm continues as in Fig. 6.5)

Figure 6.10: Graph-based backjumping learning, modifying CBJ

Deep learning

- Deep learning: recording all and only minimal conflict sets
- Example:
- Although most accurate, overhead is prohibitive: the number of conflict sets in the worst-case:

$$\binom{r}{r/2} = 2^r$$

Jumpback Learning

- Record the jumpback assignment

Example 6.7.2 For the problem and ordering of Example 6.7.1 at the first dead-end, jumpback learning will record the no-good $(x_2 = green, x_3 = blue, x_7 = red)$, since that tuple includes the variables in the jumpback set of x_1. □

procedure CONFLICT-DIRECTED-BACKJUMP-LEARNING

instantiate $x_i \leftarrow$ SELECTVALUE-CBJ
if x_i is null (no value was returned)
record a constraint prohibiting $\vec{a}_{i-1}[J_i]$ **and corresponding values**
$iprev \leftarrow i$
(algorithm continues as in Fig. 6.7)

Figure 6.11: Conflict-directed backjump-learning, modifying CBJ

Bounded and relevance-based learning

Bounding the arity of constraints recorded.

- When bound is i: i-ordered graph-based,i-order jumpback or i-order deep learning.
- Overhead complexity of i-bounded learning is time and space exponential in i.

Definition 6.7.3 (i-relevant) *A no-good is i-relevant if it differs from the current partial assignment by at most i variable-value pairs.*

Definition 6.7.4 (i'th order relevance-bounded learning) *An i'th order relevance-bounded learning scheme maintains only those learned no-goods that are i-relevant.*

Complexity of backtrack-learning (improved)

- **Theorem:** Any backtracking algorithm using graph-based learning along d has a space complexity $O(nk^{w*(d)})$ and time complexity $O(n^2(2k)^{w*(d)+1}$
- (book). Refined more: $O(n^2k^{w*(d)})$
- **Proof:** The number of deadends for each variable is $O(k^{w*(d)})$, yielding $O(nk^{w*(d)})$ deadends.There are at most *kn* values between two succesive deadends: $O(nk^{w*(d+1)})$ number of nodes in the search space. Since at most $O(2^{w*(d)})$ constraints-checks we get $\cdot$ $O(n^2(2k)^{w*(d)+1}$
- Improved more: If we have $O(nk^{w*(d)})$ leaves, we have k to n times as many internal nodes, yielding between $O(nk^{w*(d+1)})$ and nodes. $O(n^2k^{w*(d)})$

Complexity of Backtrack-Learning for CSP

- The complexity of learning along d is time and space exponential in w*(d):

The number of dead-ends is bounded by $O(nk^{w^*(d)})$

Number of constraint tests per dead-end are $O(e)$

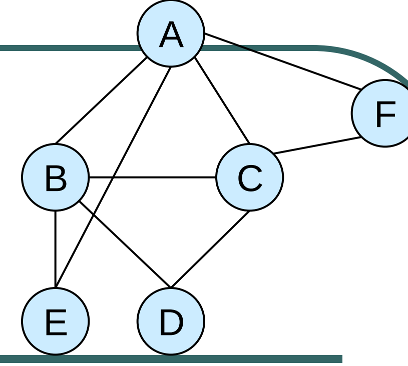
Space complexity is $O(nk^{w^*(d)})$

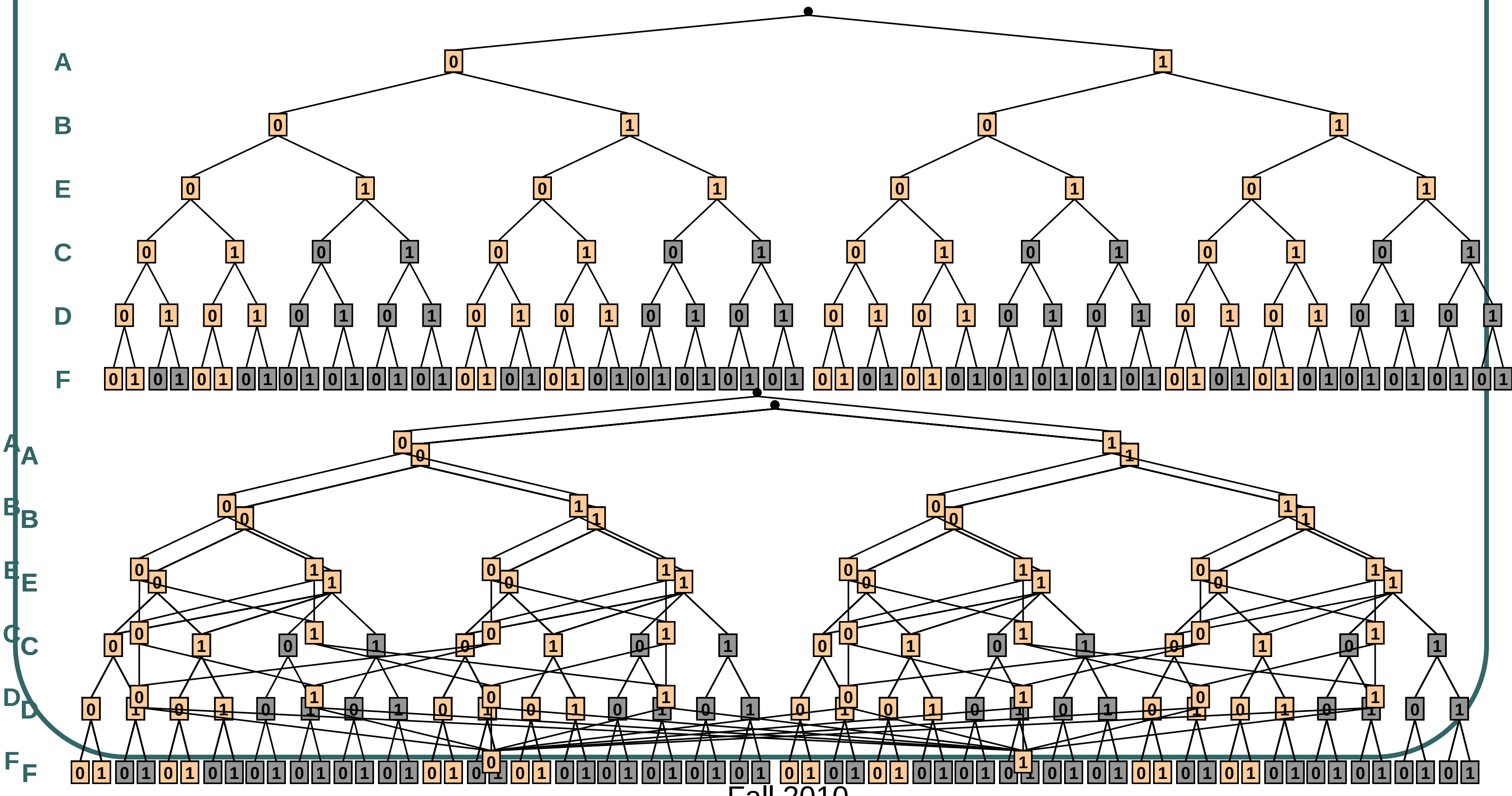
Time complexity is $O(n^2 e \cdot k^{w^*(d)})$

Learning and backjumping: $O(nmek^{w*(d)})$

m- depth of tree, e- number of constraints

Good caching:
Moving from one to all or counting





Summary:
time-space for constraint processing

- **Constraint-satisfaction**
  - Search with backjumping
    - Space: linear, Time: O(exp(logn w*))
  - Search with learning no-goods
    - time and space: O(exp(w*))
  - Variable-elimination
    - time and space: O(exp(w*))
- **Counting, enumeration**
  - Search with backjumping
    - Space: linear, Time: O(exp(n))
  - Search with no-goods caching only
    - space: O(exp(w*)) Time: O(exp(n))
  - Search with goods and no-goods learning
    - Time and space: O(exp(path-width), O(exp(log n w*))
  - Variable-elimination
    - Time and space: O(exp(w*))

Non-Systematic Randomized Learning

- Do search in a random way with interupts, restarts, unsafe backjumping, **but record conflicts**.
- Guaranteed completeness.

Look-back for SAT

- A partial assignment is a set of literals: σ
- A jumpback set if a J-clause:
- Upon a leaf deadend of *x* resolve two clauses, one enforcing x and one enforcing ~x relative to the current assignment
- A clause forces x relative to assignment σ if all the literals in the clause are negated in σ.
- Resolving the two clauses we get a nogood.
- If we identify the earliest two clauses we will find the earliest condlict.
- The argument can be extended to internal deadends.

Look-back for SAT

procedure SAT-CBJ-LEARN
Input: A CNF theory φ, assigned variables σ over $x_1, ..., x_{i-1}$, unassigned variables X,
Output: Either a solution, or a decision that the network is inconsistent.

1. $J_i \leftarrow \emptyset$
2. While $1 \le i \le n$
3. Select the next variable: $x_i \in X$, $X \leftarrow X - \{x_i\}$
4. instantiate $x_i \leftarrow$ SELECTVALUE-CBJ.
5. If x_i is null (no value returned), then
6. add J_{x_i} to φ (learning)
7. $iprev \leftarrow$ index of last variable in J_i (*backjump*)
8. $J_i \leftarrow resolve(J_i, J_{prev})$ (merge conflict sets)
9. else,
10 $i \leftarrow i + 1$ (go forward)
11. $J_i \leftarrow \emptyset$ (reset conflict set)
12. Endwhile
13. if i = 0 Return "inconsistent"
14. else, return the set of literals σ

end procedure

subprocedure SELECTVALUE-CBJ
1. If CONSISTENT$(\sigma \cup x_i)$ then return $\sigma \leftarrow \sigma \cup \{x_i\}$
2. If CONSISTENT$(\sigma \cup \neg x_i)$ then return $\sigma \leftarrow \sigma \cup \{\neg x_i\}$
3. else,
4. determine α and β the two earliest clauses forcing x_i and $\neg x_i$,
5. $J_i \leftarrow resolve(\alpha, \beta)$.
5. Return $x_i \leftarrow$ null (no consistent value)

end procedure

Figure 6.12: Algorithm SAT-CBJ-LEARN

Integration of algorithms

procedure FC-CBJ
Input: A constraint network $\mathcal{R} = (X, D, C)$.
Output: Either a solution, or a decision that the network is inconsistent.

$i \leftarrow 1$ (initialize variable counter)
call SELECTVARIABLE (determine first variable)
$D'_i \leftarrow D_i$ for $1 \leq i \leq n$ (copy all domains)
$J_i \leftarrow \emptyset$ (initialize conflict set)
while $1 \leq i \leq n$
  instantiate $x_i \leftarrow$ SELECTVALUE-FC-CBJ
  if x_i is null (no value was returned)
    $iprev \leftarrow i$
    $i \leftarrow$ latest index in J_i (backjump)
    $J_i \leftarrow J_i \cup J_{iprev} - \{x_i\}$
    reset each $D'_k, k > i$, to its value before x_i was last instantiated
  else
    $i \leftarrow i + 1$ (step forward)
    call SELECTVARIABLE (determine next variable)
    $D'_i \leftarrow D_i$
    $J_i \leftarrow \emptyset$
end while
if $i = 0$
  return "inconsistent"
else
  return instantiated values of $\{x_1, \ldots, x_n\}$
end procedure

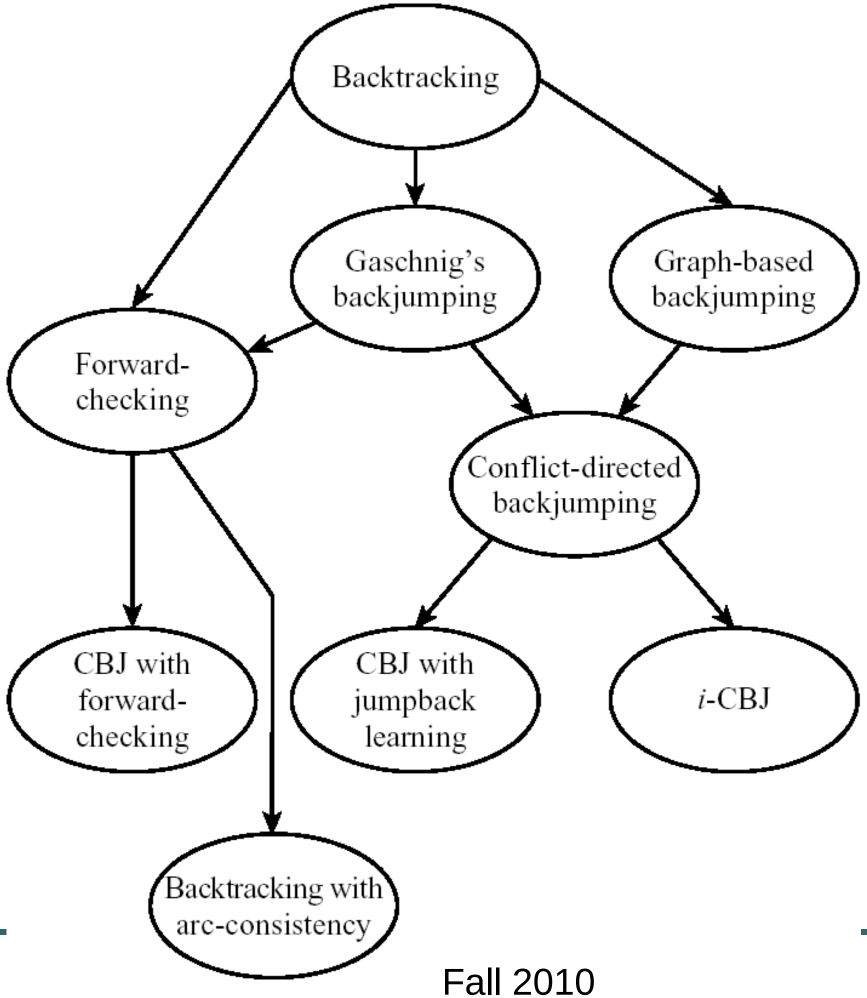
Figure 6.12: The main procedure of the FC-CBJ algorithm

subprocedure SELECTVALUE-FC-CBJ

    while D'_i is not empty
        select an arbitrary element $a \in D'_i$, and remove a from D'_i
        empty-domain $\leftarrow$ *false*
        for all k, $i < k \leq n$
            for all values b in D'_k
                if not CONSISTENT$(\vec{a}_{i-1}, x_i = a, x_k = b)$
                    let R_S be the earliest constraint causing the conflict
                    add the variables in R_S's scope S, but not x_k, to J_k
                    remove b from D'_k
            endfor
            if D'_k is empty $(x_i = a$ leads to a dead-end)
                empty-domain $\leftarrow$ *true*
        endfor
        if *empty-domain*   (don't select a)
            reset each D'_k and $j_k, i < k \leq n$, to status before a was selected
        else
            return a
    end while
    return null   (no consistent value)
end subprocedure

Figure 6.14: The SelectValue subprocedure for FC-CBJ.

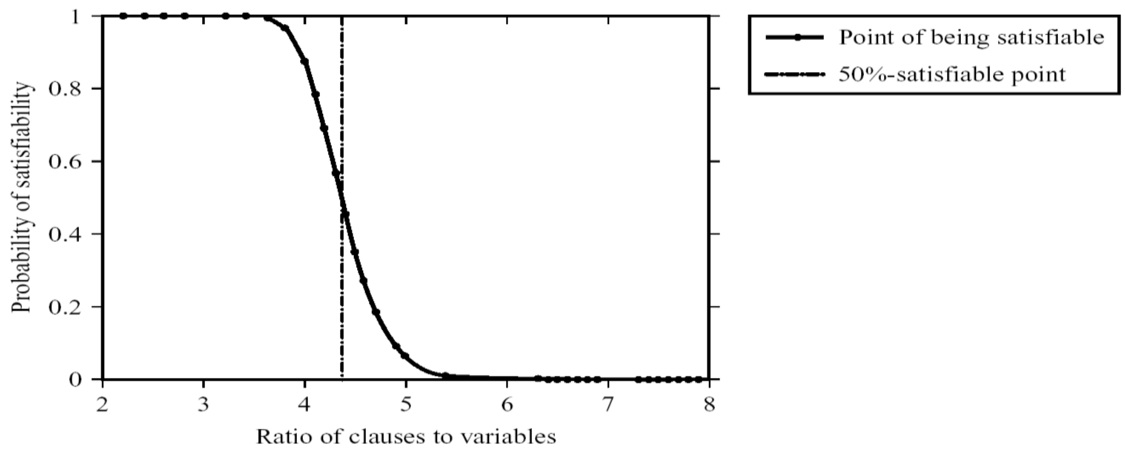
Relationships between various backtracking algrithms

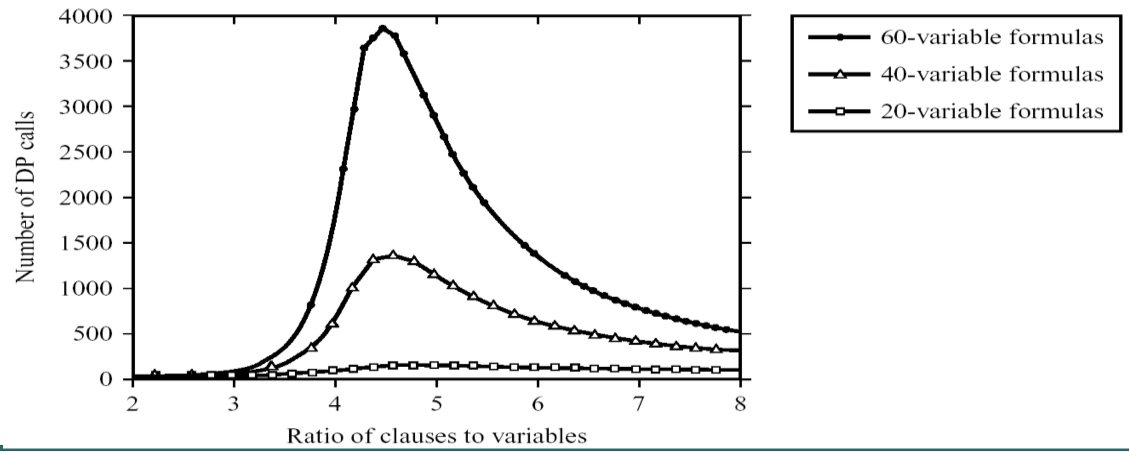


Empirical comparison of algorithms

- Benchmark instances
- Random problems
- Application-based random problems
- Generating fixed length random k-sat (n,m) uniformly at random
- Generating fixed length random CSPs
- (N,K,T,C) also arity, r.

The Phase transition (m/n)





Some empirical evaluation

- Sets 1-3 reports average over 2000 instances of random csps from 50% hardness. Set 1: 200 variables, set 2: 300, Set 3: 350. All had 3 values.:
- Dimacs problems

Algorithm	Set 1		Set 2		Set 3		ssa 038		ssa 158	
FC	207	68.5		-		-	46	14.5	52	20.0
FC+AC	40	55.4	1	0.6	1	0.4	4	3.5	18	8.2
FCr-CBJ	189	69.2	222	119.3	182	140.8	40	12.2	26	10.7
FC-CBJ+LVO	167	73.8	132	86.8	119	111.8	32	11.0	8	4.5
FC-CBJ+LRN	186	63.4	32	15.6	1	0.5	23	5.5	19	8.6
FC-CBJ+LRN+LVO	160	74.0	26	14.0	1	3.8	16	3.8	13	7.1

Figure 6.16: Empirical comparison of six selected CSP algorithms. See text for explanation. In each column of numbers, the first number indicates the number of nodes in the search tree, rounded to the nearest thousand, and final 000 omitted; the second number is CPU seconds.