Consistency algorithms

Chapter 3

Consistency methods

Approximation of inference:

Arc, path and i-consistecy

Methods that transform the original network into tighter and tighter representations

Arc-consistency

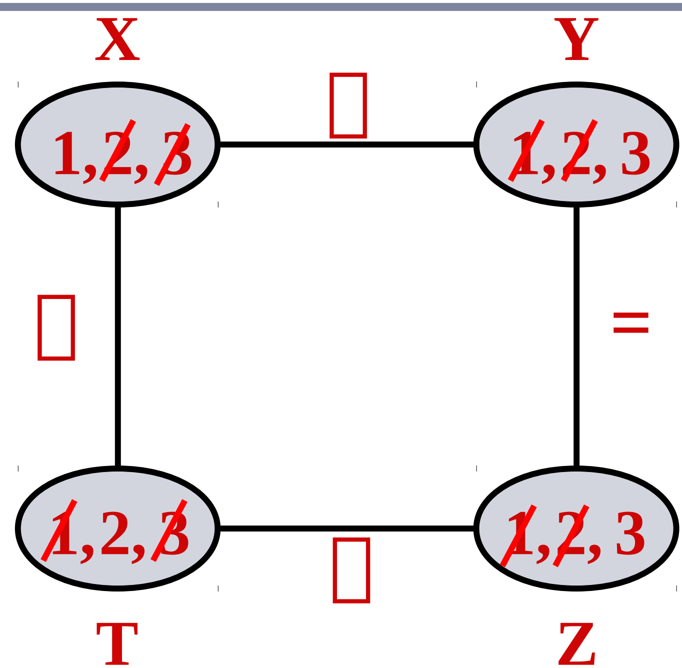
1 □ X, Y, Z, T □ 3

X □ Y

Y = Z

T □ Z

X □ T



Arc-consistency

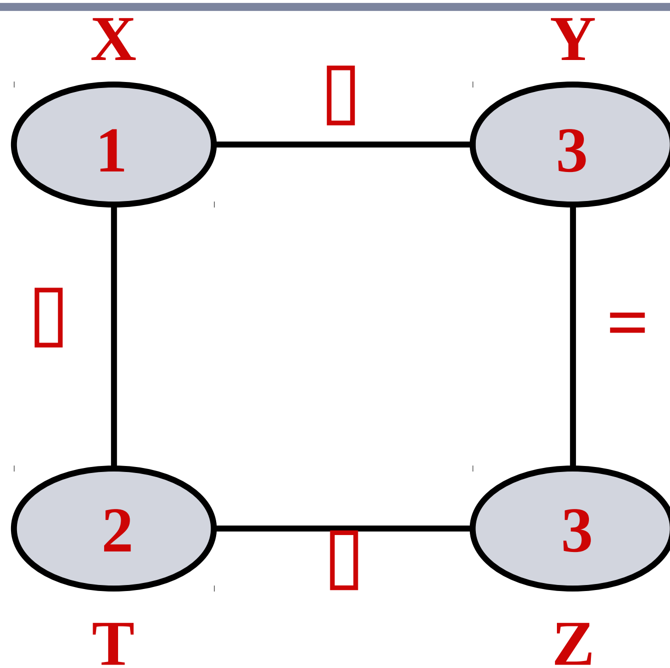
1 □ X, Y, Z, T □ 3

X □ Y

Y = Z

T □ Z

X □ T



Arc-consistency

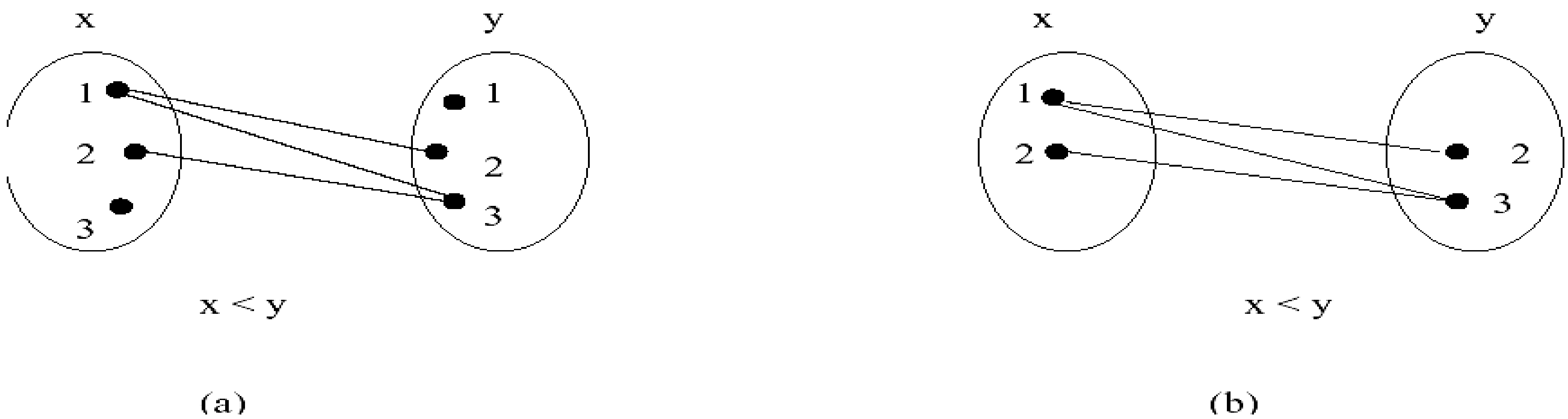


Figure 3.1: A matching diagram describing the arc-consistency of two variables x and y. In (a) the variables are not arc-consistent. In (b) the domains have been reduced, and the variables are now arc-consistent.

Definition 3.2.2 (arc-consistency) *Given a constraint network* $\mathcal{R} = (\mathcal{X}, \mathcal{D}, \mathcal{C})$, *with* $R_{ij} \in C$, *a variable* x_i *is* arc-consistent *relative to* x_j *if and only if for every value* $a_i \in D_i$ *there exists a value* $a_j \in D_j$ *such that* $(a_i, a_j) \in R_{ij}$. *The subnetwork (alternatively, the arc) defined by* $\{x_i, x_j\}$ *is arc-consistent if and only if* x_i *is arc-consistent relative to* x_j *and* x_j *is arc-consistent relative to* x_i. *A network of constraints is called* arc-consistent *iff all of its arcs (e.g., subnetworks of size 2) are arc-consistent.*

Revise for arc-consistency

REVISE$((x_i), x_j)$

input: a subnetwork defined by two variables $X = \{x_i, x_j\}$, a distinguished variable x_i, domains: D_i and D_j, and constraint R_{ij}

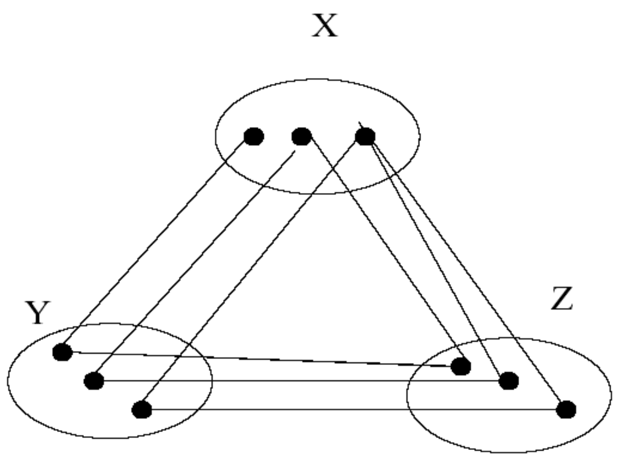
output: D_i, such that, x_i arc-consistent relative to x_j

1. **for** each $a_i \in D_i$
2. **if** there is no $a_j \in D_j$ such that $(a_i, a_j) \in R_{ij}$
3. **then** delete a_i from D_i
4. **endif**
5. **endfor**

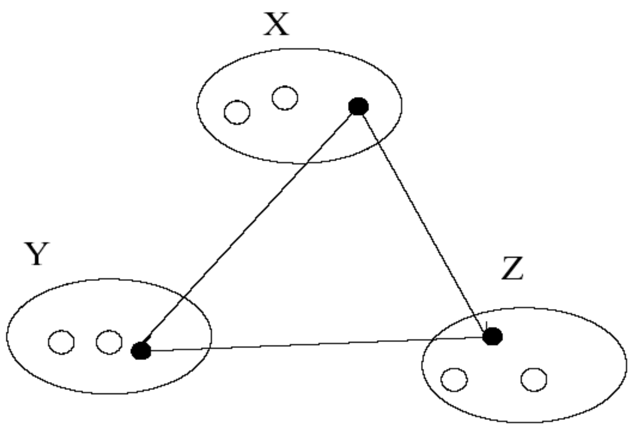
Figure 3.2: The Revise procedure

$$D_i \leftarrow D_i \cap \pi_i (R_{ij} \otimes D_j)$$

A matching diagram describing a network of constraints that is not arc-consistent (b) An arc-consistent equivalent network.



(a)



(b)

AC-1($\mathcal{R}$)

input: a network of constraints $\mathcal{R} = (X, D, C)$

output: $\mathcal{R}'$ which is the loosest arc-consistent network equivalent to $\mathcal{R}$

1. **repeat**
2. **for** every pair $\{x_i, x_j\}$ that participates in a constraint
3. Revise$((x_i), x_j)$ (or $D_i \leftarrow D_i \cap \pi_i(R_{ij} \bowtie D_j)$)
4. Revise$((x_j), x_i)$ (or $D_j \leftarrow D_j \cap \pi_j(R_{ij} \bowtie D_i)$)
5. **endfor**
6. **until** no domain is changed

Figure 3.4: Arc-consistency-1 (AC-1)

Complexity (Mackworth and Freuder, 1986): $O(enk^3)$

e = number of arcs, *n* variables, *k* values

(*ek*^2, each loop, *nk* number of loops), best-case = *ek*,

Arc-consistency is: $\Omega(ek^2)$

AC-3

AC-3($\mathcal{R}$)

input: a network of constraints $\mathcal{R} = (X, D, C)$

output: $\mathcal{R}'$ which is the largest arc-consistent network equivalent to $\mathcal{R}$

for every pair $\{x_i, x_j\}$ that participates in a constraint $R_{ij} \in \mathcal{R}$
    $queue \leftarrow queue \cup \{(x_i, x_j), (x_j, x_i)\}$
endfor
while $queue \neq \{\}$
    select and delete (x_i, x_j) from $queue$
    $Revise((x_i), x_j)$
    if $Revise((x_i), x_j)$ causes a change in D_i
        then $queue \leftarrow queue \cup \{(x_k, x_i), i \neq k\}$
    endif
endwhile

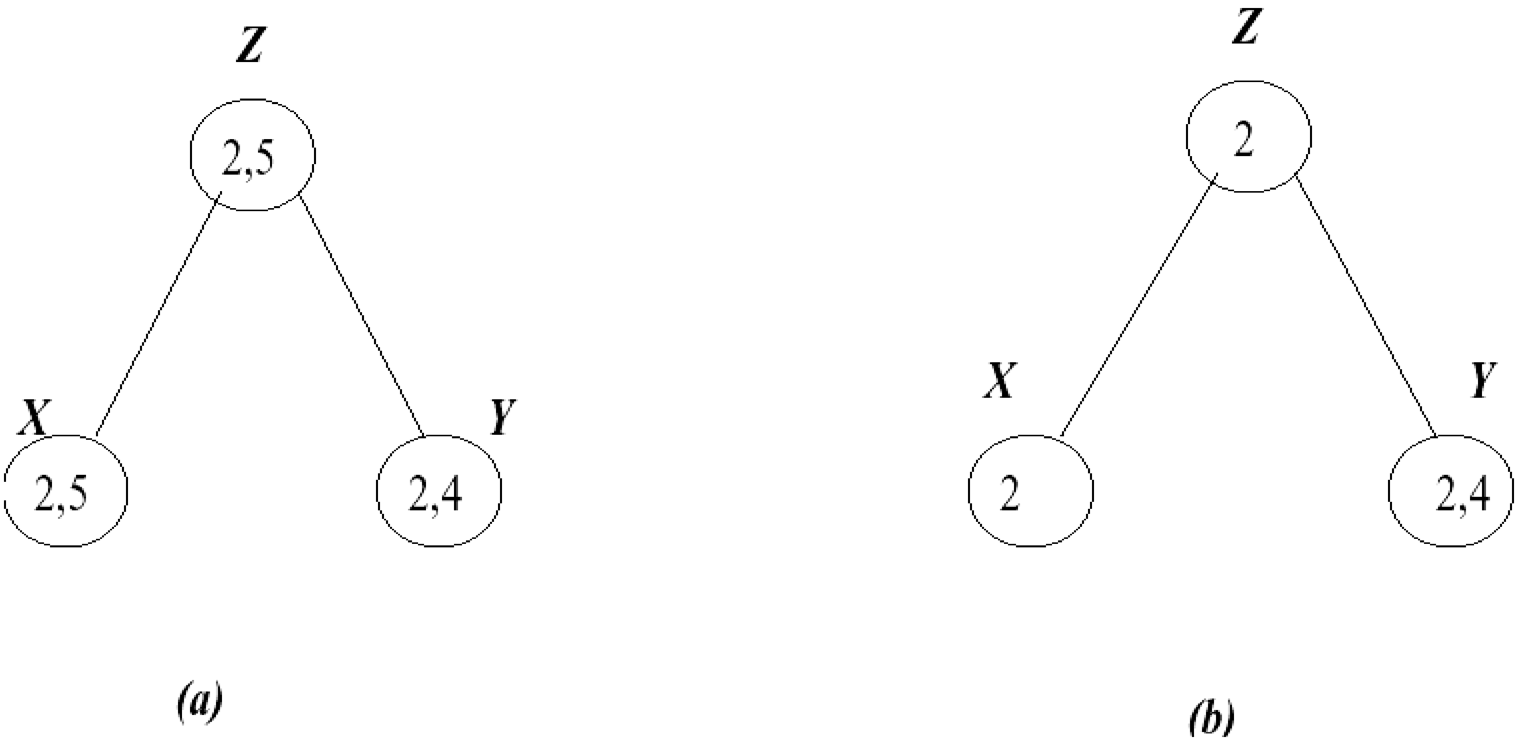
Figure 3.5: Arc-consistency-3 (AC-3)

Complexity: $O(ek^3)$ since each arc may be processed in O(2k)

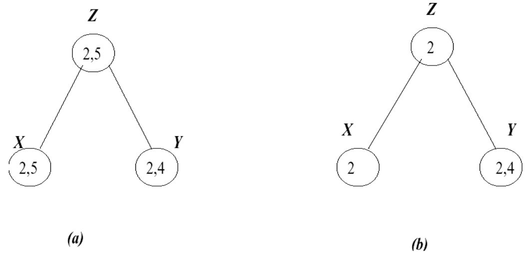
Best case O(ek),

Example: A 3 variables network with 2 constraints: z divides x and z divides y

(a) before and (b) after AC-3 is applied.



AC-4



AC-4($\mathcal{R}$)
input: a network of constraints $\mathcal{R}$
output: An arc-consistent network equivalent to $\mathcal{R}$

Initialization: $M \leftarrow \emptyset$,
    initialize $S_{(x_i,c_i)}$, $counter(i, a_i, j)$ for all R_{ij}
    for all counters
        if $counter(x_i, a_i, x_j) = 0$ (if $< x_i, a_i >$ is unsupported by x_j)
            then add $< x_i, a_i >$ to $LIST$
        endif
    endfor
while $LIST$ is not empty
    choose $< x_i, a_i >$ from LIST, remove it, and add it to M
    for each $< x_j, a_j >$ in $S_{(x_i,a_i)}$
        decrement $counter(x_j, a_j, x_i)$
        if $counter(x_j, a_j, x_i) = 0$
            then add $< x_j, a_j >$ to $LIST$
        endif
    endfor
endwhile

Figure 3.7: Arc-consistency-4 (AC-4)

Complexity: $O(ek^2)$

(Counter is the number of supports to ai in xi from xj. S_(xi,ai) is the set of pairs that (xi,ai) supports)

Example applying AC-4

Example 3.2.9 Consider the problem in Figure 3.6. Initializing the $S_{(x,a)}$ arrays (indicating all the variable-value pairs that each $< x, a >$ supports), we have : $S_{(z,2)} = \{< x, 2 >, < y, 2 >, < y, 4 >\}$, $S_{(z,5)} = \{< x, 5 >\}$, $S_{(x,2)} = \{< z, 2 >\}$, $S_{(x,5)} = \{< z, 5 >\}$, $S_{(y,2)} = \{< z, 2 >\}$, $S_{(y,4)} = \{< z, 2 >\}$.
For counters we have: $counter(x, 2, z) = 1$, $counter(x, 5, z) = 1$, $counter(z, 2, x) = 1$, $counter(z, 5, x) = 1$, $counter(z, 2, y) = 2$, $counter(z, 5, y) = 0$, $counter(y, 2, z) = 1$, $counter(y, 4, z) = 1$. (Note that we do not need to add counters between variables that are not directly constrained, such as x and y.) Finally, $List = \{< z, 5 >\}$, $M = \emptyset$. Once $< z, 5 >$ is removed from $List$ and placed in M, the counter of $< x, 5 >$ is updated to $counter(x, 5, z) = 0$, and $< x, 5 >$ is placed in $List$. Then, $< x, 5 >$ is removed from $List$ and placed in M. Since the only value it supports is $< z, 5 >$ and since $< z, 5 >$ is already in M, the $List$ remains empty and the process stops. □

Distributed arc-consistency (Constraint propagation)

Implement AC-1 distributedly.

$$D_i \leftarrow D_i \cap \pi_i(R_{ij} \otimes D_j)$$

Node x_j sends the message to node x_i

$$h_i^j \leftarrow \pi_i(R_{ij} \otimes D_j)$$

Node x_i updates its domain:

$$D_i \leftarrow D_i \cap \pi_i(R_{ij} \otimes D_j) =$$

Messages can be sent asynchronously or scheduled in a topological order

$$D_i \leftarrow D_i \cap h_i^j$$

Exercise: make the following network arc-consistent

Draw the network's primal and dual constraint graph

Network =

Domains {1,2,3,4}

Constraints: y < x, z < y, t < z, f<t, x<=t+1, Y<f+2

Arc-consistency Algorithms

AC-1: brute-force, distributed $O(nek^3)$

AC-3, queue-based $O(ek^3)$

AC-4, context-based, optimal $O(ek^2)$

AC-5,6,7,…. Good in special cases

Important: applied at every node of search

(*n* number of variables, *e*=#constraints, *k*=domain size)

Mackworth and Freuder (1977,1983), Mohr and Anderson, (1985)…

Using constraint tightness in analysis

t = number of tuples bounding a constraint

AC-1: brute-force, $O(nek^3)$ $O(nekt)$

AC-3, queue-based $O(ek^3)$ $O(ekt)$

AC-4, context-based, optimal $O(et)$

AC-5,6,7,…. Good in special cases

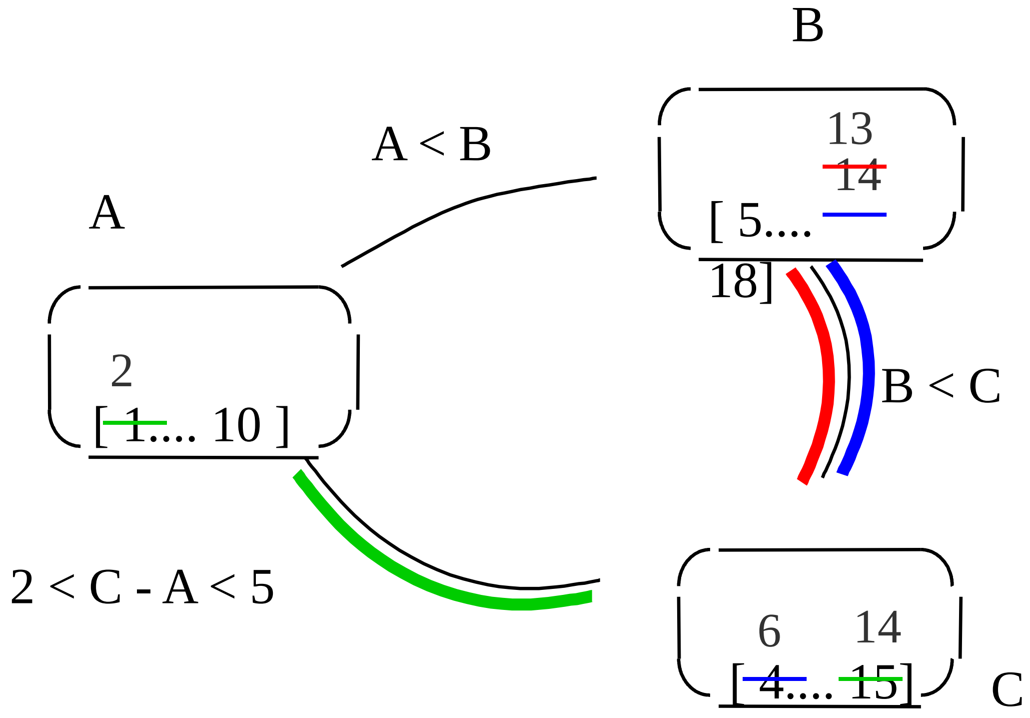
Important: applied at every node of search

(n number of variables, e=#constraints, k=domain size)

Mackworth and Freuder (1977,1983), Mohr and Anderson, (1985)…

Constraint checking

- Arc-consistency



1- B: [5 .. 14]

C: [6 .. 15]

2- A: [2 .. 10]

C: [6 .. 14]

3- B: [5 .. 13]

Is arc-consistency enough?

Example: a triangle graph-coloring with 2 values.

Is it arc-consistent?

Is it consistent?

It is not path, or 3-consistent.

Path-consistency

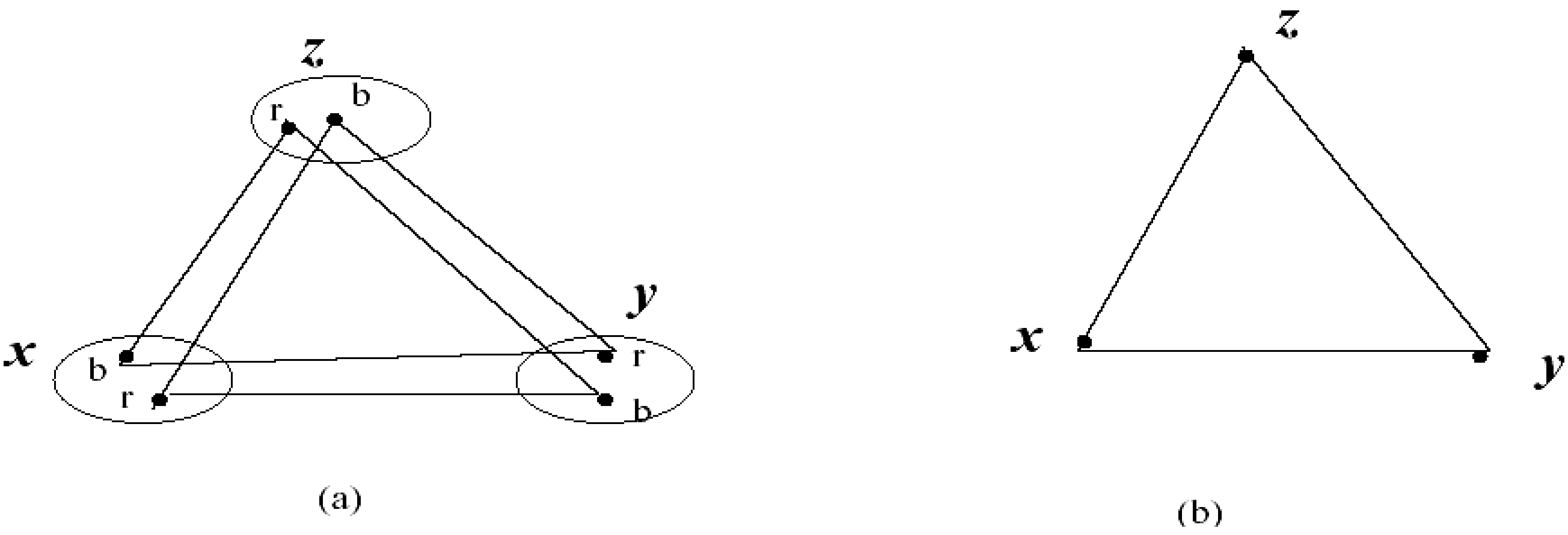


Figure 3.8: (a) The matching diagram of a 2-value graph coloring problem. (b) Graphical picture of path-consistency using the matching diagram.

Path-consistency

Click to edit Master text styles

Second level

Definition 3.3.2 (Path-consistency) *Given a constraint network* $\mathcal{R} = (X, D, C)$, *a*

Alternatively, a binary constraint R_{ij} *is path-consistent relative to* x_k *iff for every pair* $(a_i, a_j), \in R_{ij}$, *where* a_i *and* a_j *are from their respective domains, there is a value* $a_k \in D_k$ *s.t.* $(a_i, a_k) \in R_{ik}$ *and* $(a_k, a_j) \in R_{kj}$. *A subnetwork over three variables* $\{x_i, x_j, x_k\}$ *is path-consistent iff for any permutation of* (i, j, k), R_{ij} *is path consistent relative to* x_k. *A network is path-consistent iff for every* R_{ij} *(including universal binary relations) and for every* $k \neq i, j$ R_{ij} *is path-consistent relative to* x_k.

Revise-3

REVISE-3$((x, y), z)$

input: a three-variable subnetwork over (x, y, z), R_{xy}, R_{yz}, R_{xz}.
output: revised R_{xy} path-consistent with z.
1. **for** each pair $(a, b) \in R_{xy}$
2. **if** no value $c \in D_z$ exists such that $(a, c) \in R_{xz}$ and $(b, c) \in R_{yz}$
3. **then** delete (a, b) from R_{xy}.
4. **endif**
5. **endfor**

Figure 3.9: Revise-3

$$R_{ij} \leftarrow R_{ij} \cap \pi_{ij}(R_{ik} \otimes D_k \otimes R_{kj})$$

Complexity: O(k^3)

Best-case: O(t)

Worst-case O(tk)

PC-1

PC-1($\mathcal{R}$)

input: a network $\mathcal{R} = (X, D, C)$.
output: a path consistent network equivalent to $\mathcal{R}$.

1. **repeat**
2. **for** $k \leftarrow 1$ to n
3. **for** $i, j \leftarrow 1$ to n
4. $R_{ij} \leftarrow R_{ij} \cap \pi_{ij}(R_{ik} \bowtie D_k \bowtie R_{kj})$/* $(Revise-3((i,j),k))$
5. **endfor**
6. **endfor**
7. **until** no constraint is changed.

Figure 3.10: Path-consistency-1 (PC-1)

Complexity: $O(n^5 k^5)$

O(n^3) triplets, each take O(k^3) steps O(n^3 k^3)

Max number of loops: O(n^2 k^2) .

PC-2

PC-3($\mathcal{R}$)

input: a network $\mathcal{R} = (X, D, C)$.
output: $\mathcal{R}'$ a path consistent network equivalent to $\mathcal{R}$.

1. $Q \leftarrow \{(i,k,j) \mid 1 \leq i < j \leq n, 1 \leq k \leq n, k \neq i, k \neq j\}$
2. **while** Q is not empty
3. select and delete a 3-tuple (i,k,j) from Q
4. $R_{ij} \leftarrow R_{ij} \cap \pi_{ij}(R_{ik} \bowtie D_k \bowtie R_{kj})$ /* (Revise-3($(i,j),k$))
5. **if** R_{ij} changed then
6. $Q \leftarrow Q \cup \{(l,i,j)(l,j,i) \mid 1 \leq l \leq n, l \neq i, l \neq j\}$
7. **endwhile**

Figure 3.11: Path-consistency-3 (PC-3)

Complexity: $O(n^3 k^5)$

Optimal PC-4: $O(n^3 k^3)$

(each pair deleted may add: 2n-1 triplets, number of pairs: O(n^2 k^2) size of Q is O(n^3 k^2), processing is O(k^3))

Example: before and after path-consistency

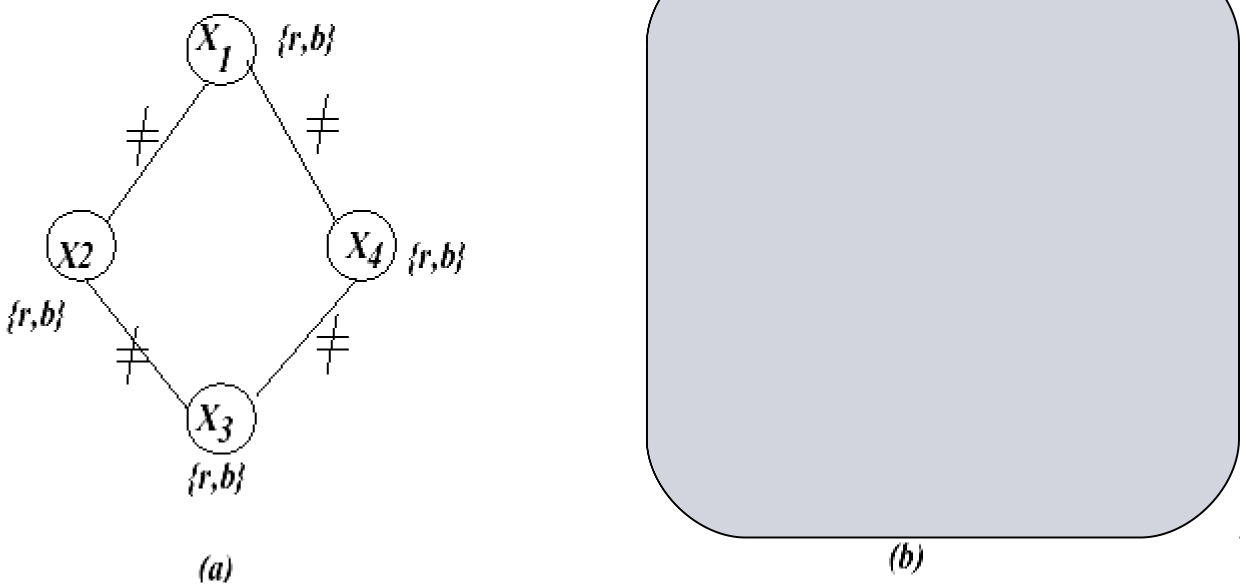


Figure 3.12: A graph-coloring graph (a) before path-consistency (b) after path-consistency

PC-1 requires 2 processings of each arc while PC-2 may not

Can we do path-consistency distributedly?

Example: before and after path-consistency

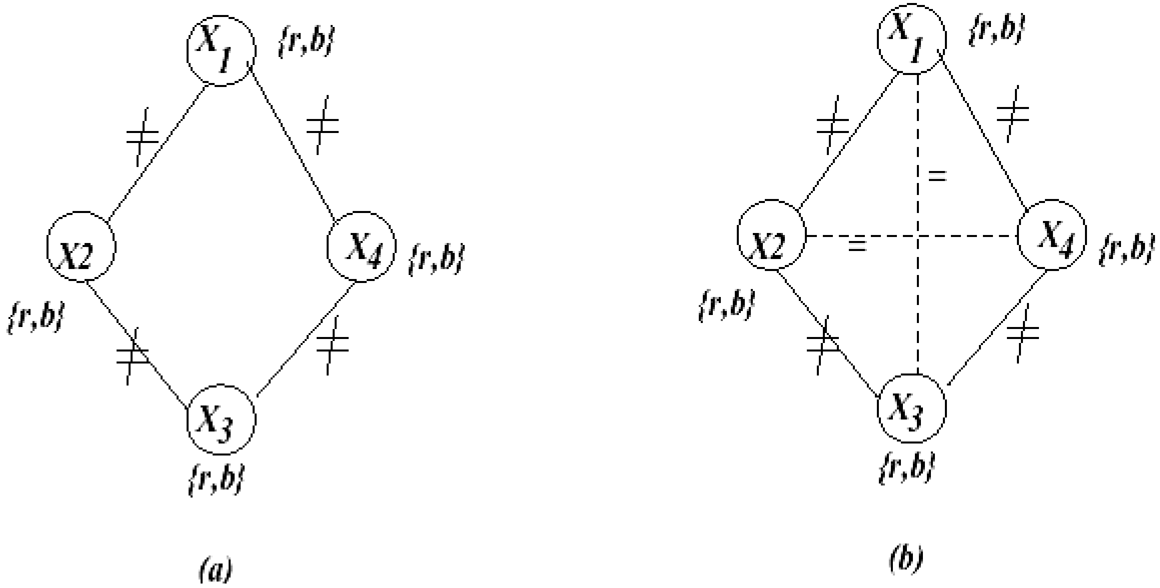


Figure 3.12: A graph-coloring graph (a) before path-consistency (b) after path-consistency

PC-1 requires 2 processings of each arc while PC-2 may not

Can we do path-consistency distributedly?

Path-consistency Algorithms

Apply Revise-3 (O(k^3)) until no change

$$R_{ij} \leftarrow R_{ij} \cap \pi_{ij}(R_{ik} \otimes D_k \otimes R_{kj})$$

Path-consistency (3-consistency) adds binary constraints.

PC-1: $O(n^5k^5)$

PC-2: $O(n^3k^5)$

PC-4 optimal: $O(n^3k^3)$

I-consistency

Click to edit Master text styles
- Second level
  - Third level
    - Fourth level
      - Fifth level

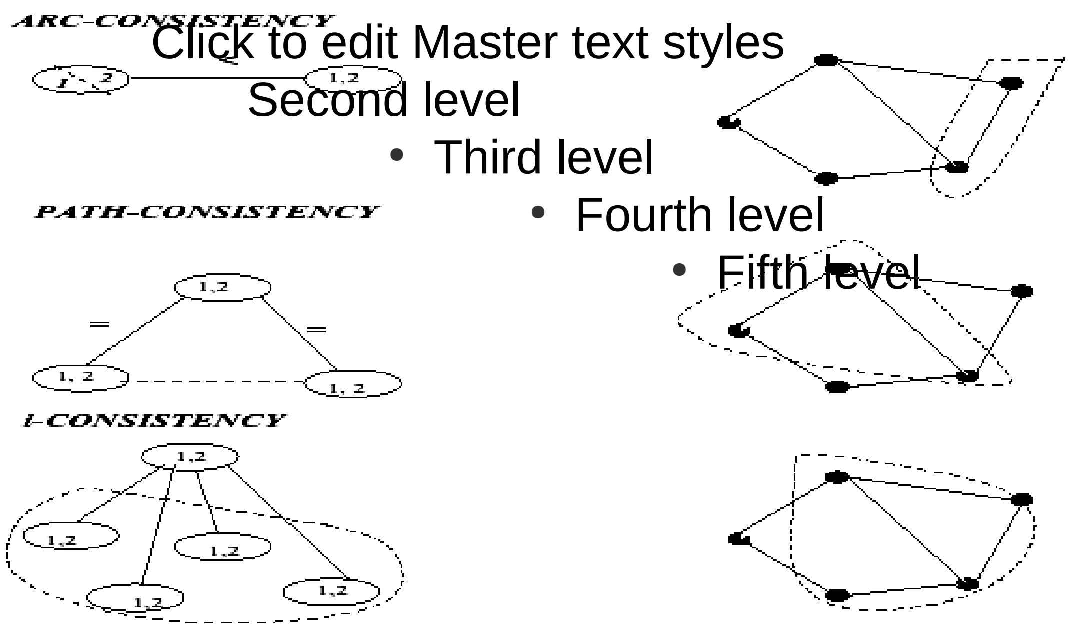


Figure 3.17: The scope of consistency enforcing: (a) arc-consistency, (b) path-consistency, (c) i-consistency

Higher levels of consistency, global-consistency

Definition:

- Fourth level
  - Fifth level

A network is i-consistent *iff given any consistent instantiation of any* $i-1$ *distinct variables, there exists an instantiation of any* i*th variable such that the* i *values taken together satisfy all of the constraints among the* i *variables. A network is* strongly i-consistent *iff it is* j*-consistent for all* $j \leq i$*. A strongly* n*-consistent network, where* n *is the number of variables in the network, is called* globally consistent.

Revise-i

$\text{REVISE-}i(\{x_1, x_2, ..., x_{i-1}\}, x_i)$

input: a network $\mathcal{R} = (X, D, C)$
output: a constraint R_S, $S = \{x_1, ..., x_{i-1}\}$ i-consistent relative to x_i.
1. **for** each instantiation $\bar{a}_{i-1} = (< x_1, a_1 >, < x_2, a_2 >, ..., < x_{i-1}, a_{i-1} >)$ do,
2. **if** no value of $a_i \in D_i$ exists s.t. $(\bar{a}_{i-1}, a_i)$ is consistent
   then delete $\bar{a}_{i-1}$ from R_S
   (Alternatively, let $\mathcal{S}$ be the set of all subsets of $\{x_1, ..., x_i\}$ that contain x_i and appear as scopes of constraints of $\mathcal{R}$, then
   $R_S \leftarrow R_S \cap \pi_S(\bowtie_{S' \subseteq \mathcal{S}} R_{S'}))$
3. **endfor**

Figure 3.14: Revise-i

Complexity: for binary constraints $O(k^i)$

For arbitrary constraints: $O((2k)^i)$

4-queen example

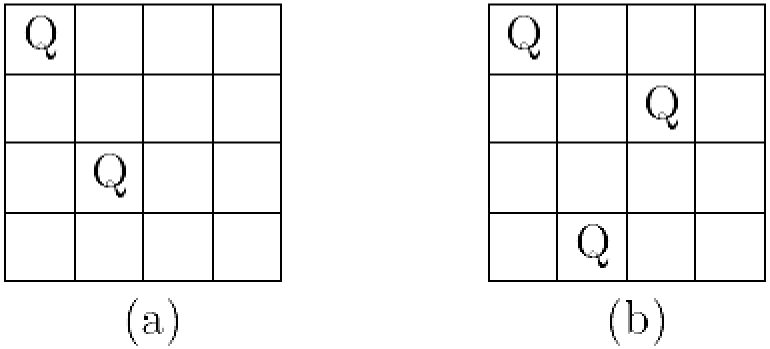


Figure 3.13: (a) Not 3-consistent; (b) Not 4-consistent

I-consistency

I-CONSISTENCY($\mathcal{R}$)
input: a network $\mathcal{R}$.
output: an i-consistent network equivalent to $\mathcal{R}$.

```
1.   repeat
2.        for every subset S ⊆ X of size i − 1, and for every x_i, do
3.             let 𝒮 be the set of all subsets in of {x_1, ..., x_i} scheme(ℛ)
               that contain x_i
4.             R_S ← R_S ∩ π_S(⋈_{S'∈𝒮} R_{S'}) ( this is Revise-i(S, x_i))
6.        endfor
7.   until no constraint is changed.
```

Figure 3.15: i-consistency-1

Theorem 3.4.3 (complexity of i-consistency) *The time and space complexity of brute-force i-consistency $O(2^i(nk)^{2i})$ and $O(n^i k^i)$, respectively. A lower bound for enforcing i-consistency is $\Omega(n^i k^i)$.* □

I-consistency

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

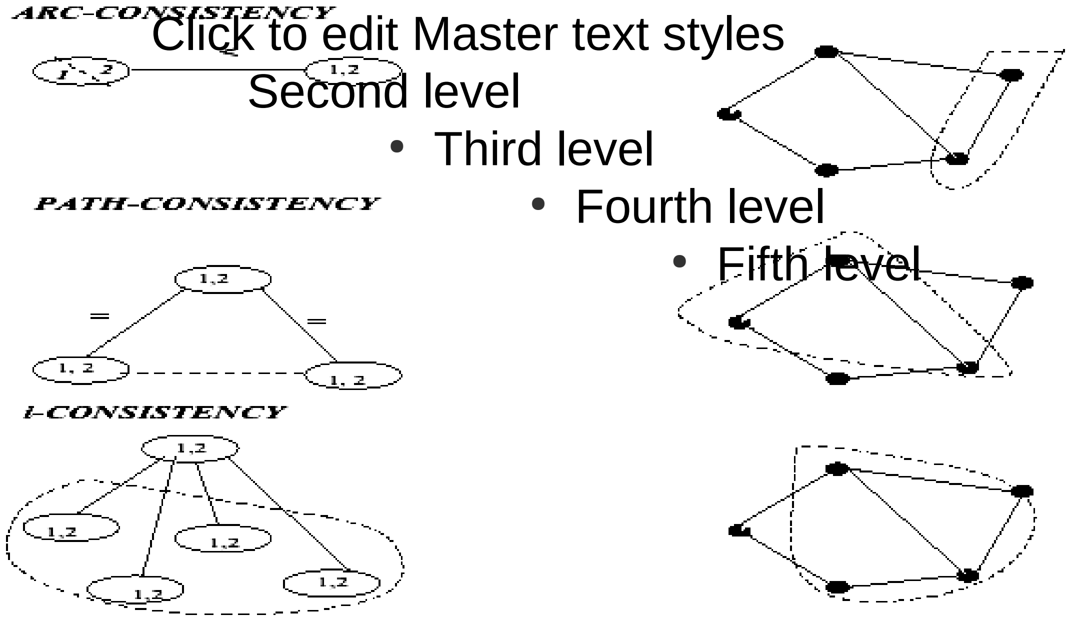


Figure 3.17: The scope of consistency enforcing: (a) arc-consistency, (b) path-consistency, (c) i-consistency

Arc-consistency for non-binary constraints: Generalized arc-consistency

Definition 3.5.1 (generalized arc-consistency) *Given a constraint network $\mathcal{R} = (\mathcal{X}, \mathcal{D}, \mathcal{C})$, with $R_S \in C$, a variable x is arc-consistent relative to R_S if and only if for every value $a \in D_x$ there exists a tuple $t \in R_S$ such that $t[x] = a$. t can be called a support for a. The constraint R_S is called arc-consistent iff it is arc-consistent relative to each of the variables in its scope and a constraint network is arc-consistent if all its constraints are arc-consistent.*

$$D_x \leftarrow D_x \cap \pi_x (R_S \otimes D_{S-\{x\}})$$

Complexity: O(t k), t bounds number of tuples.

Relational arc-consistency:

$$R_{S-\{x\}} \leftarrow \pi_{S-\{x\}} (R_S \otimes D_x)$$

Examples of generalized arc-consistency

x+y+z <= 15 and z >= 13 implies

x<=2, y<=2

Example of relational arc-consistency

$$A \wedge B \rightarrow G, \neg G, \Rightarrow \neg A \vee \neg B$$

More arc-based consistency

Global constraints: e.g., all-different constraints

Special semantic constraints that appears often in practice and a specialized constraint propagation. Used in constraint programming.

Bounds-consistency: pruning the boundaries of domains

Sudoku – Constraint Satisfaction

- **Constraint**
- **Propagation**

- **Inference**

		2	4		6			
8	6	5	1			2		
	1				8	6		9
9				4		8	6	
	4	7				1	9	
	5	8		6				3
4		6	9				7	
		9			4	5	8	1
			3		2	9		

- **Variables: empty slots**
- **Domains = {1,2,3,4,5,6,7,8,9}**
- **Constraints:**
  - **27 all-different**

Each row, column and major block must be alldifferent

“Well posed” if it has unique solution: 27 constraints

Example for alldiff

A = {3,4,5,6}

B = {3,4}

C= {2,3,4,5}

D= {2,3,4}

E = {3,4}

F= {1,2,3,4,5,6}

Alldiff (A,B,C,D,E}

Arc-consistency does nothing

Apply GAC to sol(A,B,C,D,E,F)?

 A = {6}, F = {1}….

Alg: bipartite matching kn^1.5

(Lopez-Ortiz, et. Al, IJCAI-03 pp 245 (A fast and simple algorithm for bounds consistency of alldifferent constraint)

Global constraints

Alldifferent

Sum constraint (variable equal the sum of others)

Global cardinality constraint (a value can be assigned a bounded number of times to a set of variables)

The cummulative constraint (related to scheduling tasks)

Bounds consistency

Definition 3.5.4 (bounds consistency) *Given a constraint C over a scope S and domain constraints, a variable $x \in S$ is bounds-consistent relative to C if the value $min\{D_x\}$ (respectively, $max\{D_x\}$) can be extended to a full tuple t of C. We say that t supports $min\{D_x\}$. A constraint C is* bounds-consistent *if each of its variables is bounds-consistent.*

Click to edit Master text styles
- Second level
  - Third level
    - Fourth level
      - Fifth level

Bounds consistency

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Example 3.5.5 Consider the constraint problem with variables $x_1, \ldots x_6$, each with domains $1, \ldots, 6$, and constraints

$$C_1 : x_4 \geq x_1 + 3, \quad C_2 : x_4 \geq x_2 + 3, \quad C_3 : x_5 \geq x_3 + 3, \quad C_4 : x_5 \geq x_4 + 1,$$

$$C_5 : alldifferent\{x_1, x_2, x_3, x_4, x_5\}$$

The constraints are not bounds consistent. For example, the minimum value 1 in the domain of x_4 does not have support in constraint C_1 as there is no corresponding value for x_1 that satisfies the constraint. Enforcing bounds consistency using constraints C_1 through C_4 reduces the domains of the variables as follows: $D_1 = \{1, 2\}$, $D_2 = \{1, 2\}$, $D_3 = \{1, 2, 3\}$ $D_4 = \{4, 5\}$ and $D_5 = \{5, 6\}$. Subsequently, enforcing bounds consistency using constraints C_5 further reduces the domain of C to $D_3 = \{3\}$.Now constraint C_3 is no longer bound consistent. Reestablishing bounds consistency causes the domain of x_5 to be reduced to $\{6\}$. Is the resulting problem already arc-consistent? □

Boolean constraint propagation

(A V ~B) and (B)

B is arc-consistent relative to A but not vice-versa

Arc-consistency by resolution:

res((A V ~B),B) = A

Given also (B V C), path-consistency:

res((A V ~B),(B V C) = (A V C)

Relational arc-consistency rule = unit-resolution

$$A \wedge B \rightarrow G, \neg G, \Rightarrow \neg A \vee \neg B$$

Constraint propagation for Boolean constraints: Unit propagation

Click to edit Master text styles
Second level
- Third level
  - Fourth level
    - Fifth level

Procedure UNIT-PROPAGATION
Input: A cnf theory, φ, $d = Q_1, ..., Q_n$.
Output: An equivalent theory such that every unit clause does not appear in any non-unit clause.
1. queue = all unit clauses.
2. **while** queue is not empty, do.
3. $T \leftarrow$ next unit clause from Queue.
4. **for** every clause β containing T or $\neg T$
5. **if** β contains T delete β (subsumption elimination)
6. **else**, For each clause $\gamma = \textit{resolve}(\beta, T)$.
   if γ, the resolvent, is empty, the theory is unsatisfiable.
7. **else**, add the resolvent γ to the theory and delete β.
   if γ is a unit clause, add to Queue.
8. **endfor**.
9. **endwhile**.

Theorem 3.6.1 *Algorithm* UNIT-PROPAGATION *has a linear time complexity.*

Click to edit Master text styles
Second level
- Third level
  - Fourth level

Example (if there is time)

— M: The unicorn is mythical

– I: The unicorn is immortal

– L: The unicorn is mammal

– H: The unicorn is horned

– G: The unicorn is magical

$(M \rightarrow I) \wedge (\neg M \rightarrow (\neg I \wedge L)) \wedge ((I \vee L) \rightarrow H) \wedge (H \rightarrow G)$

A Logic Puzzle IV

• Is the unicorn mythical? Is it magical? Is it horned?

$(M \rightarrow I) \wedge (\neg M \rightarrow (\neg I \wedge L)) \wedge ((I \vee L) \rightarrow H) \wedge (H \rightarrow G) \vdash$

$(\neg M \vee I) \wedge (M \vee (\neg I \wedge L)) \wedge ((I \vee L) \rightarrow H) \wedge (H \rightarrow G) \vdash$

$(\neg M \vee I) \wedge (M \vee \neg I) \wedge (M \vee L) \wedge ((I \vee L) \rightarrow H) \wedge (H \rightarrow G) \vdash$

$(I \vee L) \wedge ((I \vee L) \rightarrow H) \wedge (H \rightarrow G) \vdash H \wedge G$

• Hence, the unicorn is not necessarily mythical, but it is horned and

magical !

Consistency for numeric constraints (Gausian elimination)

$$x \in [1,10],\ y \in [5,15],$$

$$x + y = 10$$

$$arc-consistency \Rightarrow x \in [1,5],\ y \in [5,9]$$

$$by-adding-x+y=10, -y \le -5$$

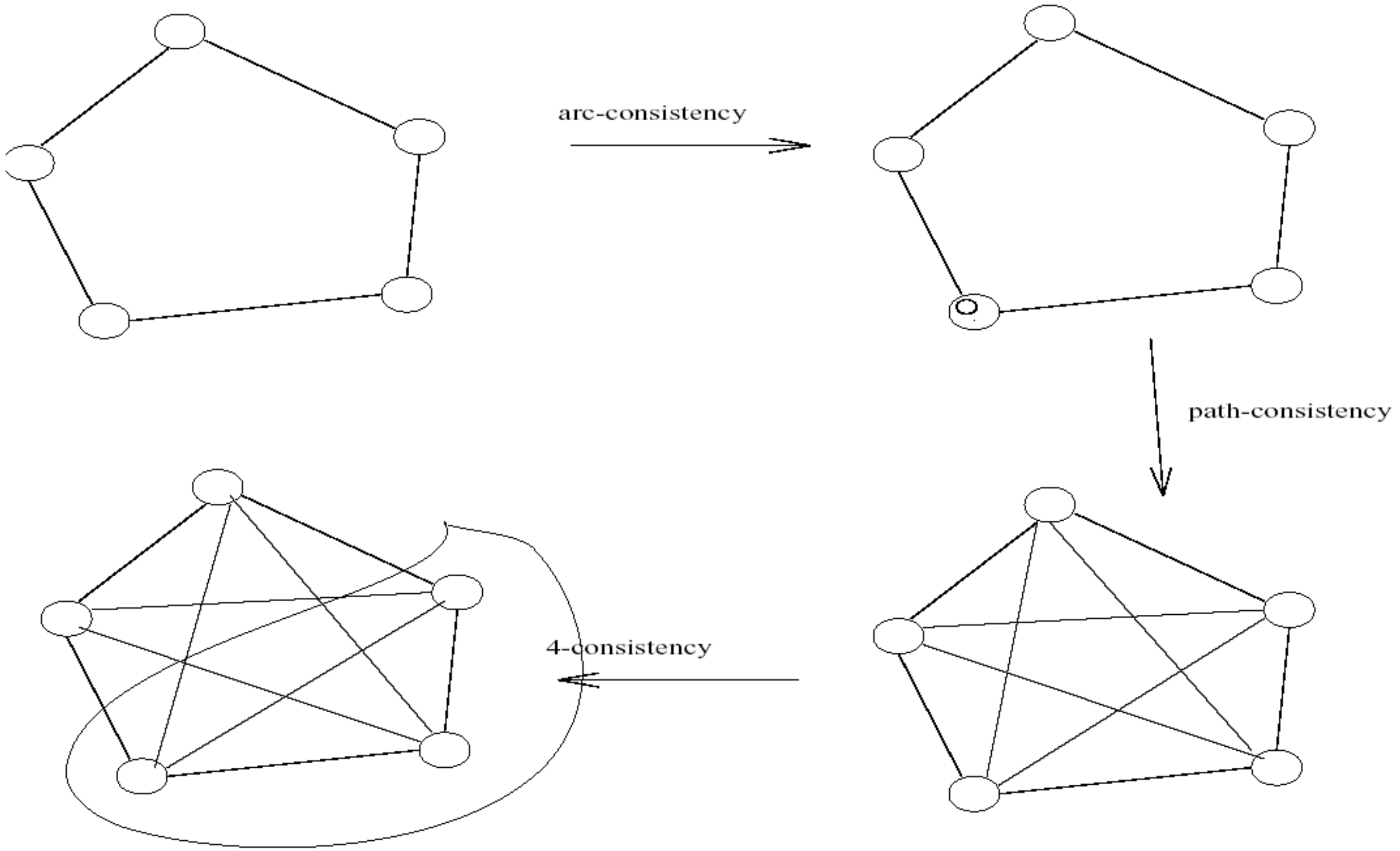
$$z \in [-10,10],$$

$$y + z \le 3$$

$$path-consistency \Rightarrow x - z \ge 7$$

$$obtained-by-adding, x+y=10, -y-z \ge -3$$

Changes in the network graph as a result of arc-consistency, path-consistency and 4-consistency.



Distributed arc-consistency (Constraint propagation)

Implement AC-1 distributedly.

$$D_i \leftarrow D_i \cap \pi_i(R_{ij} \otimes D_j)$$

Node x_j sends the message to node x_i

$$h_i^j \leftarrow \pi_i(R_{ij} \otimes D_j)$$

$$D_i \leftarrow D_i \cap h_i^j$$

Node x_i updates its domain:

Relational and generalized arc-consistency can be implemented distributedly: sending messages between constraints over the dual graph

$$R_{S-\{x\}} \leftarrow \pi_{S-\{x\}}(R_S \otimes D_x)$$

Relational Arc-consistency

The message that R2 sends to R1 is

$$h_i^j \leftarrow \pi_{l_{ij}}(R_i \bowtie (\bowtie_{k \in ne(i)} h_k^i))$$

R1 updates its relation and domains and sends messages to neighbors

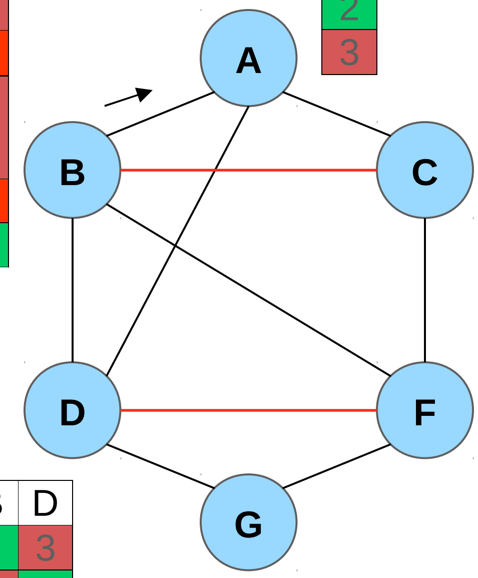
$$D_i \leftarrow D_i \cap (\bowtie_{k \in ne(i)} D_k^i)$$

R_2

A	B
1	2
1	3
2	1
2	3
3	1
3	2

R_1

A
1
2
3



R_3

A	C
1	2
3	2

R_5

B	C	F
1	2	3
3	2	1

R_4

A	B	D
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

R_6

D	F	G
1	2	3
2	1	3

Distributed Relational Arc-Consistency

DRAC can be applied to the dual problem of any constraint network:

$$h_i^j \leftarrow \pi_{l_{ij}}(R_i \bowtie (\bowtie_{k \in ne(i)} h_k^i)) \qquad (1)$$

$$R_i \leftarrow R_i \cap (\bowtie_{k \in ne(i)} h_k^i) \qquad (2)$$

R_1

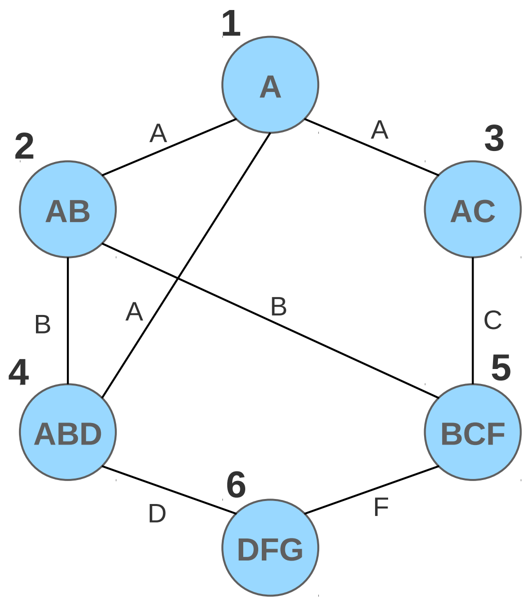
A
1
2
3

R_2

A	B
1	2
1	3
2	1
2	3
3	1
3	2

R_3

A	C
1	2
3	2



R_4

A	B	D
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

R_5

B	C	F
1	2	3
3	2	1

R_6

D	F	G
1	2	3
2	1	3

Iteration 1

R_1

A
1
2
3

h_2^1

A
1
2
3

h_3^1

A
1
3

h_4^1

A
1
2
3

h_5^2

B
1
3

h_4^2

A	B
1	2
1	3
2	1
2	3
3	1
3	2

h_3^2

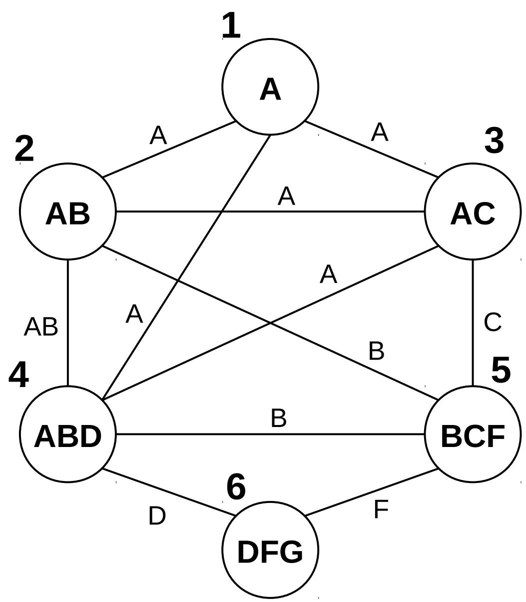
A
1
3

h_1^2

A
1
2
3

R_2

A	B
1	2
1	3
2	1
2	3
3	1
3	2



R_3

A	C
1	2
3	2

h_1^3

A
1
2
3

h_2^3

A
1
2
3

h_4^3

A
1
2
3

h_5^3

C
2

h_6^4

D
1
2

h_5^4

B
1
3

h_3^4

A
1
3

h_2^4

A	B
1	2
1	3
2	1
2	3
3	1
3	2

h_1^4

A
1
2
3

R_4

A	B	D
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

R_5

B	C	F
1	2	3
3	2	1

h_2^5

B
1
2
3

h_3^5

C
2

h_4^5

B
1
2
3

h_6^5

F
1
3

R_6

D	F	G
1	2	3
2	1	3

h_4^6

D
1
2
3

h_5^6

F
1
3

$$R_i \leftarrow R_i \bigcap (\bowtie_{k \in ne(i)} h_k^i) \quad (2)$$

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Iteration 1

R_1

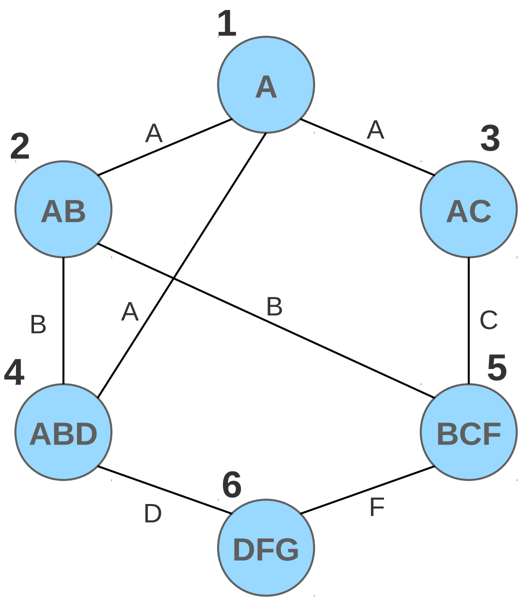
A
1
3

R_2

A	B
1	3
2	1
2	3
3	1

R_3

A	C
1	2
3	2



R_4

A	B	D
1	3	2
2	3	1
3	1	2
3	2	1

R_5

B	C	F
1	2	3
3	2	1

R_6

D	F	G
2	1	3

$$h_i^j \leftarrow \pi_{l_{ij}}(R_i \bowtie (\bowtie_{k \in ne(i)} h_k^i)) \quad (1)$$

Iteration 2

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

R_1

A
1
3

h_2^1

A
1
2
3

h_3^1

A
1
3

h_4^1

A
1
2
3

h_5^2

B
1
3

h_4^2

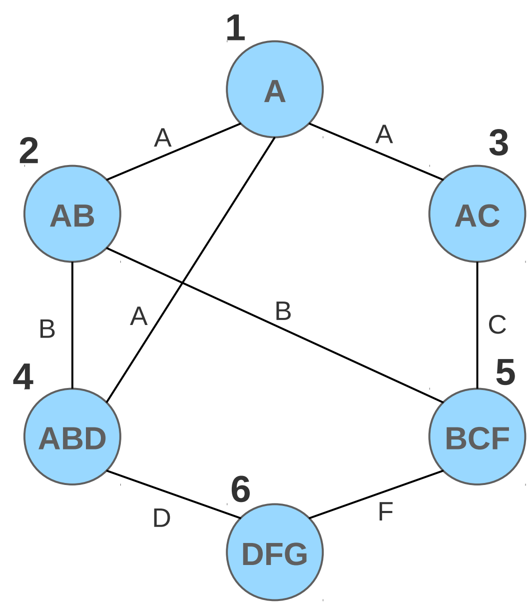
B
1
2
3

h_1^2

A
1
3

R_2

A	B
1	3
2	1
2	3
3	1



R_3

A	C
1	2
3	2

h_1^3

A
1
3

h_5^3

C
2

h_6^4

D
2

h_2^4

B
1
3

h_1^4

A
1
3

R_4

A	B	D
1	3	2
2	3	1
3	1	2
3	2	1

R_5

B	C	F
1	2	3
3	2	1

h_2^5

B
1
3

h_3^5

C
2

h_6^5

F
1

R_6

D	F	G
2	1	3

h_4^6

D
1
2

h_5^6

F
1
3

Iteration 2

$$R_i \leftarrow R_i \bigcap (\bowtie_{k\in ne(i)} h_k^i) \qquad (2)$$

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

R_1

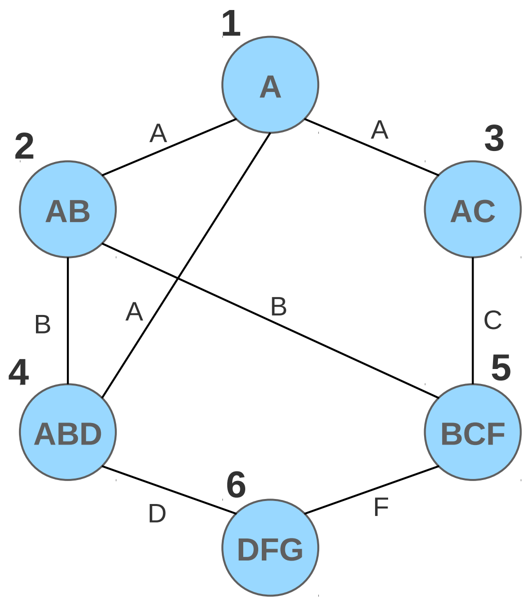
A
1
3

R_2

A	B
1	3
3	1

R_3

A	C
1	2
3	2



R_4

A	B	D
1	3	2
3	1	2

R_5

B	C	F
3	2	1

R_6

D	F	G
2	1	3

$$h_i^j \leftarrow \pi_{l_{ij}}(R_i \bowtie (\bowtie_{k \in ne(i)} h_k^i)) \quad (1)$$

Click to edit Master text styles
Second level
- Third level
  - Fourth level
    - Fifth level

Iteration 3

R_1

A
1
3

h_2^1

A
1
3

h_3^1

A
1
3

h_4^1

A
1
3

h_5^2

B
3

h_4^2

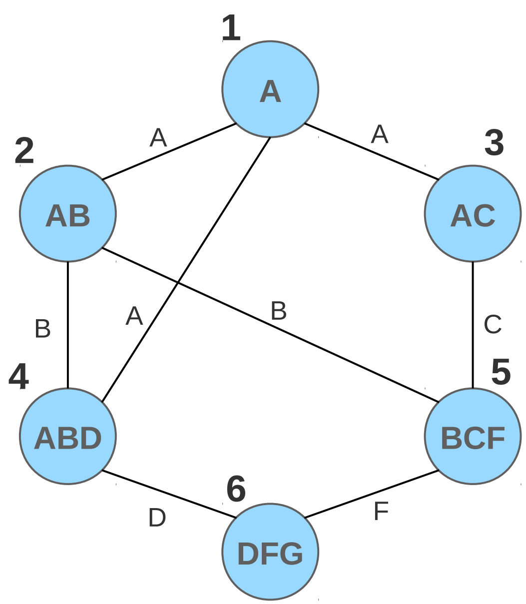
B
1
3

h_1^2

A
1
3

R_2

A	B
1	3
3	1



R_3

A	C
1	2
3	2

h_1^3

A
1
3

h_5^3

C
2

h_6^4

D
2

h_2^4

B
1
3

h_1^4

A
1
3

R_4

A	B	D
1	3	2
3	1	2

R_5

B	C	F
3	2	1

h_2^5

B
1
3

h_3^5

C
2

h_6^5

F
1

R_6

D	F	G
2	1	3

h_4^6

D
2

h_5^6

F
1

$$R_i \leftarrow R_i \cap (\bowtie_{k \in ne(i)} h_k^i) \quad (2)$$

Iteration 3

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

R_1

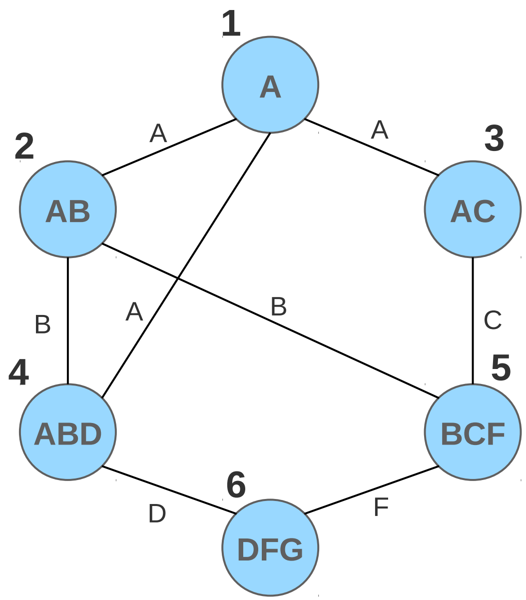
A
1
3

R_2

A	B
1	3

R_3

A	C
1	2
3	2



R_4

A	B	D
1	3	2
3	1	2

R_5

B	C	F
3	2	1

R_6

D	F	G
2	1	3

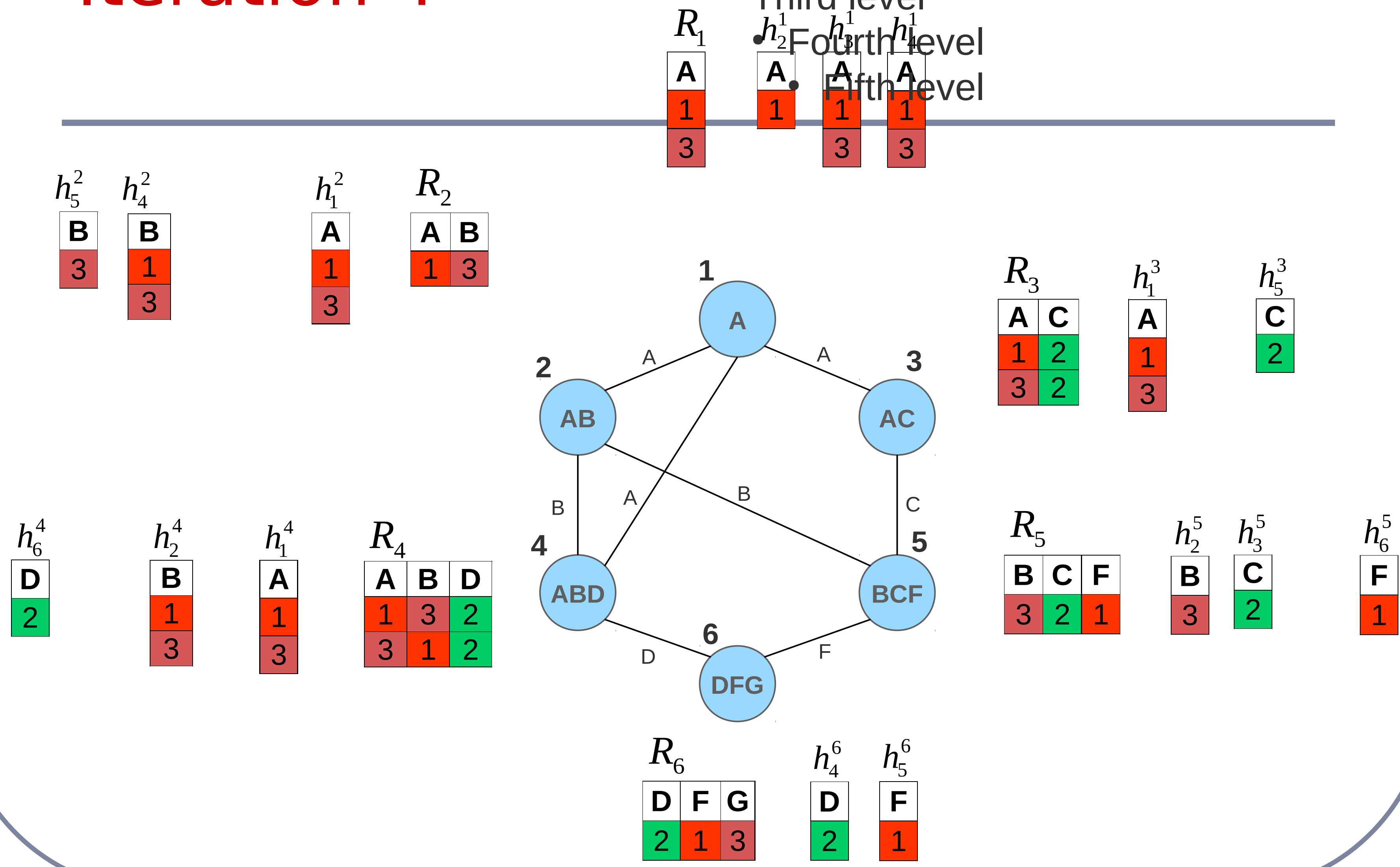
Iteration 4

$$h_i^j \leftarrow \pi_{l_{ij}}\left(R_i \bowtie \left(\bowtie_{k \in ne(i)} h_k^i\right)\right) \quad (1)$$

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level



$$R_i \leftarrow R_i \bigcap (\bowtie_{k \in ne(i)} h_k^i) \quad (2)$$

Iteration 4

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

R_1

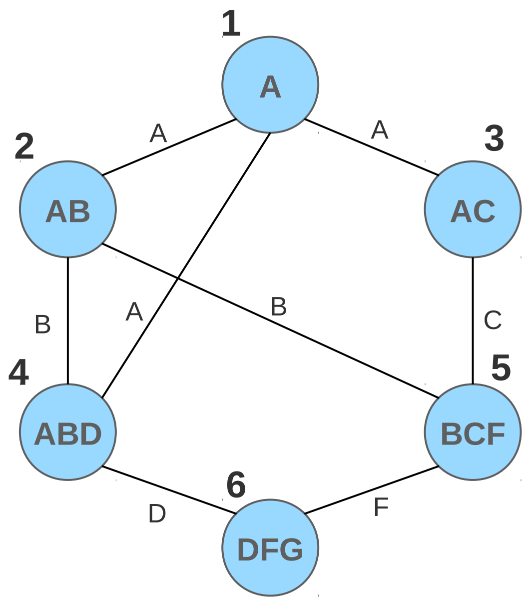
A
1

R_2

A	B
1	3

R_3

A	C
1	2
3	2



R_4

A	B	D
1	3	2

R_5

B	C	F
3	2	1

R_6

D	F	G
2	1	3

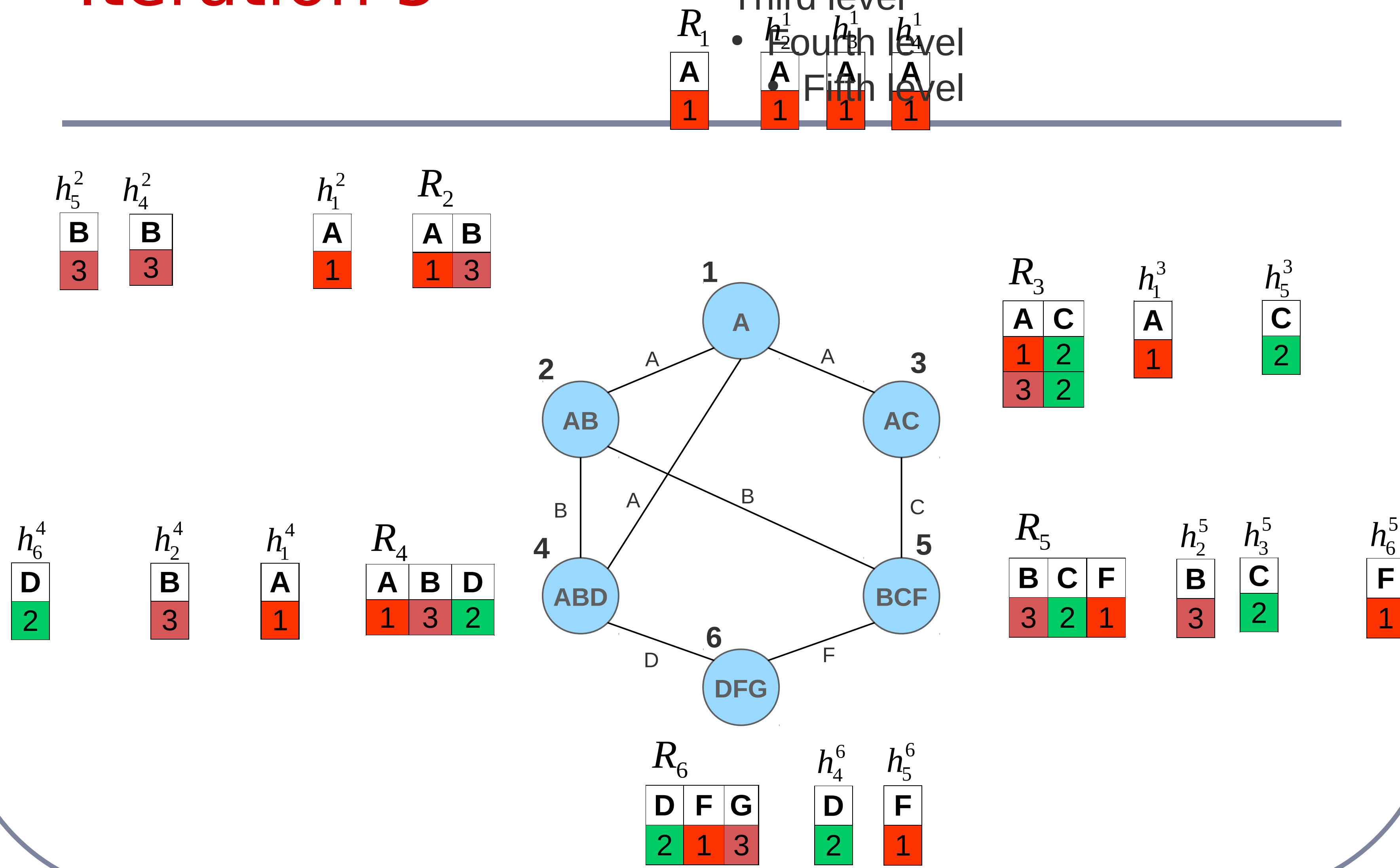
Iteration 5

$$h_i^j \leftarrow \pi_{l_{ij}}(R_i \bowtie (\bowtie_{k \in ne(i)} h_k^i)) \quad (1)$$

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level



Iteration 3

$$R_i \leftarrow R_i \cap (\bowtie_{k \in ne(i)} h_k^i) \quad (2)$$

Click to edit Master text styles
Second level
- Third level
  - Fourth level
    - Fifth level

R_1

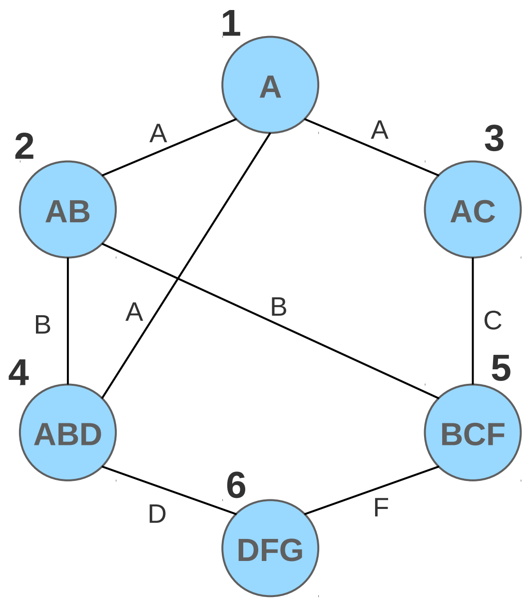
A
1

R_2

A	B
1	3

R_3

A	C
1	2



R_4

A	B	D
1	3	2

R_5

B	C	F
3	2	1

R_6

D	F	G
2	1	3

Tractable classes

Theorem 3.7.1 *1. The consistency of binary constraint networks having no cycles can be decided by arc-consistency,*

2. The consistency of binary constraint networks with bi-valued domains can be decided by path-consistency,

3. The consistency of Horn cnf theories can be decided by unit propagation.

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level