Stochastic greedy local search

Chapter 7

ICS-275

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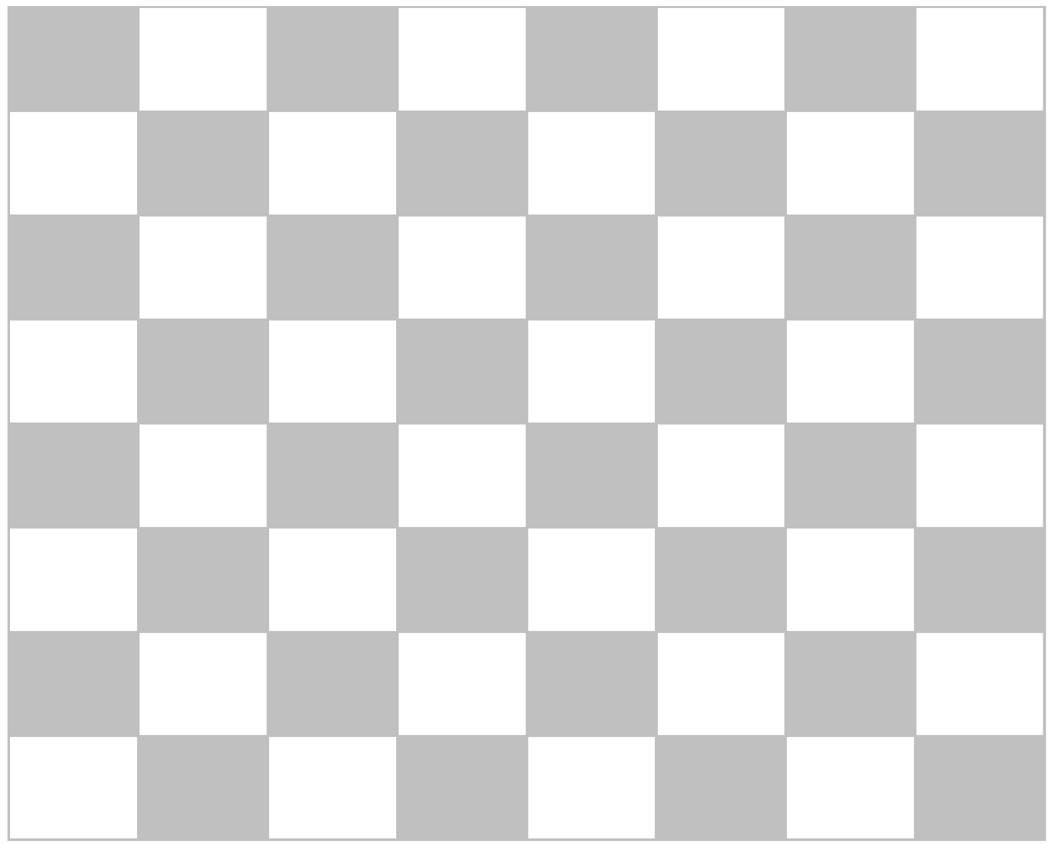
Example: 8-queen problem



Main elements

- Choose a full assignment and iteratively improve it towards a solution
- Requires a cost function: number of unsatisfied constraints or clasuses. Neural networks use energy minimization
- Drawback: local minimas
- Remedy: introduce a random element
- Cannot decide inconsistency

Algorithm Stochastic Local search (SLS)

Procedure SLS

Input: A constraint network $\mathcal{R} = (X, D, C)$, number of tries MAX_TRIES. A cost function.

Output: A solution iff the problem is consistent, "false" otherwise.

1. **for** i=1 to MAX_TRIES
   - **initialization: let** $\bar{a} = (a_1, ..., a_n)$ be a random initial assignment to all variables.
   - **repeat**
     (a) **if** $\bar{a}$ is consistent, return $\bar{a}$ as a solution.
     (b) **else** let $Y = \{< x_i, a'_i >\}$ be the set of variable-value pairs that when x_i is assigned a'_i, give a maximum improvement in the cost of the assignment; pick a pair $< x_i, a'_i > \in Y$,
     $\bar{a} \leftarrow (a_1, ..., a_{i-1}, a'_i, a_{i+1}, ..., a_n)$ (just flip a_i to a'_i).
   - **until** the current assignment cannot be improved.
2. **endfor**
3. return **false**

Example: CNF

Example 7.1 Consider the formula $\varphi = \{(\neg C)(\neg A \vee \neg B \vee C)(\neg A \vee D \vee E)(\neg B \vee \neg C)\}$. Assume that in the initial assignment all variables are assigned the value "1". This assignment violates two clauses, the first and the last, so the cost is 2. Next we see that flipping A, E or D will not remove any inconsistency. Flipping C to "0" will satisfy the two violated clauses but will violate the clause $(\neg A \vee \neg B \vee C)$, yielding a cost of 1. Flipping B to $\neg B$ will remove one inconsistency and has a cost of 1 as well. If we flip C to $\neg C$, and subsequently flipping $B$to $\neg B$ yields a cost of 0 – and a solution. □

- Example: z divides y,x,t z = {2,3,5}, x,y = {2,3,4}, t={2,5,6}

Heuristics for improving local search

- Plateau search: at local minima continue search sideways.
- Constraint weighting: use weighted cost function
  - The cost C_i is 1 if no violation. At local minima increase the weights of violating constraints.

$$F(\bar{a}) = \sum w_i C_i(\bar{a})$$

- Tabu search:
  - prevent backwards moves by keeping a list of assigned variable-values. Tie-breaking rule may be conditioned on historic information: select the value that was flipped least recently
- Automating Max-flips:
  - Based on experimenting with a class of problems
  - Given a progress in the cost function, allow the same number of flips used up to current progress.

Random walk strategies

- Combine random walk with greediness
  - At each step:
    - choose randomly an unsatisfied clause.
    - with probability p flip a random variable in the clause, with (1-p) do a greedy step minimizing the breakout value: the number of new constraints that are unsatisfied

Figure 7.2: Algorithm WalkSAT

Procedure WalkSAT
Input: A network $\mathcal{R} = (X, D, C)$, number of flips MAX_FLIPS, MAX_TRIES, probability p.
Output: True iff the problem is consistent, false otherwise.

1. For i= 1 to MAX_TRIES do
2. Compare best assignment with $\bar{a}$ and retain the best.
   (a) **start** with a random initial assignment $\bar{a}$.
   (b) **for** i=1 to MAX_FLIPS
   - **if** $\bar{a}$ is a solution, return **true** and $\bar{a}$.
   - **else,**
     i. **pick** a violated constraint C, randomly
     ii. **choose** with probability p a variable-value pair $< x, a' >$ for $x \in scope(C)$, or, with probability $1-p$, choose a variable-value pair $< x, a' >$ that minimizes the number of new constraints that break when the value of x is changed to a', (minus 1 if the current constraint is satisfied).
     iii. Change x's value to a'.
3. **endfor**
4. return **false** and the best current assignment.

Example of walkSAT:
start with assignment of true to all vars

Example 7.2 Following our earlier example 7.1.1, we will first select an unsatisfied clause, such as $(\neg B \vee \neg C)$, and then select a variable. If we try to minimize the number of additional constraints that would be broken, we will select B and flip its value. Subsequently, the only unsatisfied clause is $\neg C$ which is selected and flipped. □

$$(\neg C), (\neg A \vee \neg B \vee C)(\neg A \vee D \vee E)(\neg B \vee \neg C)$$

Simulated Annealing (Kirkpatric, Gellat and Vecchi (1983))

- Pick randomly a variable and a value and compute delta: the change in the cost function when the variable is flipped to the value.
- If change improves execute it,
- Otherwise it is executed with probability $e^{-\frac{\delta}{T}}$ where T is a temperature parameter.
- The algorithm will converge if T is reduced gradually.

Properties of local search

- Guarantee to terminate at local minima
- Random walk on 2-sat is guaranteed to converge with probability 1 after N^2 steps, when N is the number of variables.
- Proof:
  - A random assignment is on the average N/2 flips away from a satisfying assignment.
  - There is at least ½ chance that a flip of a 2-clause will reduce the distance to a given satisfying assignment by 1.
  - Random walk will cover this distance in N^2 steps on the average.
- Analysis breaks for 3-SAT
- Empirical evaluation shows good performance compared with complete algorithms (see chapter and numerous papers)

Hybrids of local search and Inference

- We can use exact hybrids of search+inference and replace search by SLS (Kask and Dechter 1996)
  - Good when cutset is small
- The effect of preprocessing by constraint propagation on SLS (Kask and Dechter 1995)
  - Great improvement on structured problems
  - Not so much on uniform problems

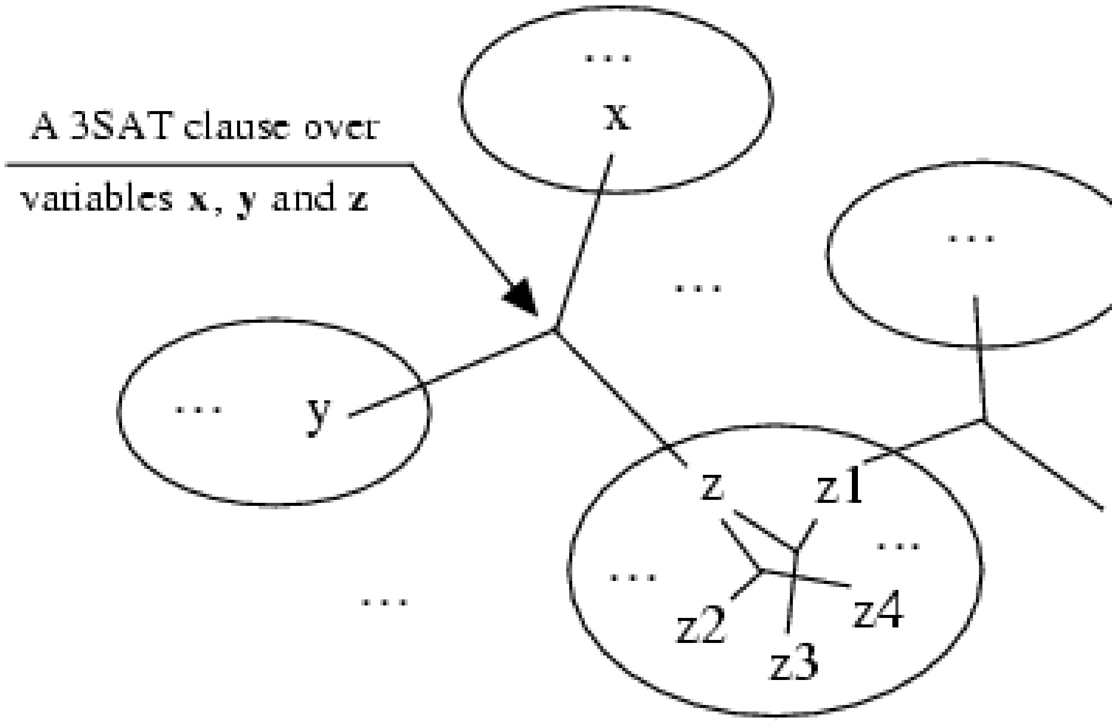
SLS and Local Consistency

- **Structured** (hierarchical 3SAT cluster structures) vs. **(uniform) random**.

Basic scheme :

- Apply preprocessing (resolution, path consistency)
- Run SLS
- Compare against SLS alone

SLS and Local Consistency



http://www.ics.uci.edu/%7Ecsp/r34-gsat-local-consistency.pdf

SLS and Local Consistency

Solvable 3SAT cluster structures, 100 instances, MaxFlips = 512K 5 variables per cluster, 50 clusters, 200 clauses between clusters Restricted Bound-3 Resolution : only original clauses resolved Running times, number of flips and clauses added are given as an average per problem solved										
	Before Resolution			After Resolution						
C/cluster	Solved	Time	Flips	Solved	RBR-3 Time	Total Time	Flips	New Clauses	DP
30	100	0.52 sec	4.5K	100	3.6 sec	3.7 sec	189	1736	1.03 sec
31	100	0.71	5.1K	100	3.88	3.91	176	1731	1.04
32	100	1.03	8.4K	100	4.16	4.20	162	1722	1.09
33	100	1.54	12K	100	4.36	4.39	155	1708	1.11
34	100	3.44	26K	100	4.66	4.70	151	1690	1.15
35	100	6.38	49K	100	4.92	4.95	140	1668	1.18
36	90	21.7	161K	100	5.23	5.26	135	1640	1.19
37	41	35.5	252K	100	5.42	5.45	131	1609	1.23
38	3	28.4	202K	100	5.94	5.97	125	1574	1.27
39	0	-	-	100	5.95	5.98	121	1540	1.29
40	0	-	-	100	6.13	6.17	115	1503	1.29

Table 1: Bound-3 Resolution and GSAT

http://www.ics.uci.edu/%7Ecsp/r34-gsat-local-consistency.pdf

SLS and Local Consistency

N=100, K=8, T=32/64, 200 instances, MaxFlips = 512K								
C	Solvable	Algorithm	Solved	Tries	Flips	PPC Time	Total Time	BJ-DVO
265	88.5 %	GSAT	139	336	147K	0 sec	36 sec	19 min
		PPC + GSAT	152	292	140K	8 sec	66 sec	
270	66 %	GSAT	78	406	191K	0 sec	45 sec	33 min
		PPC + GSAT	83	381	195K	14 sec	92 sec	
N=30, K=64, T=2048/4096, 100 instances, MaxFlips = 128K, C_{crit}=180?								
163		PPC + GSAT	56	276	59K	56 sec	153 sec	*
		GSAT	58	247	53K	0 sec	89 sec	*

Table 2: Partial Path Consistency and GSAT

Uniform random 3SAT, N=600, C=2550, 100 instances, MaxFlips = 512K					
Algorithm	Solved	Tries	Flips	BR-3 Time	Total Time
GSAT	36	63	176K	0 sec	15.3 sec
BR-3 + GSAT	31	45	125K	0.3 sec	15.0 sec

Table 3: Bound-3 Resolution and GSAT

SLS and Local Consistency

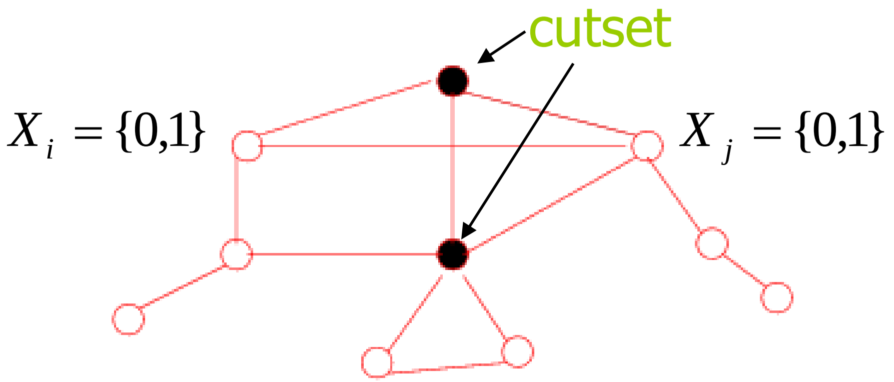
Summary:

- For structured problems, enforcing local consistency will improve SLS
- For uniform CSPs, enforcing local consistency is not cost effective: performance of SLS is improved, but not enough to compensate for the preprocessing cost.

SLS and Cutset Conditioning

Background:

- Cycle cutset technique improves backtracking by conditioning only on cutset variables.



SLS and Cutset Conditioning

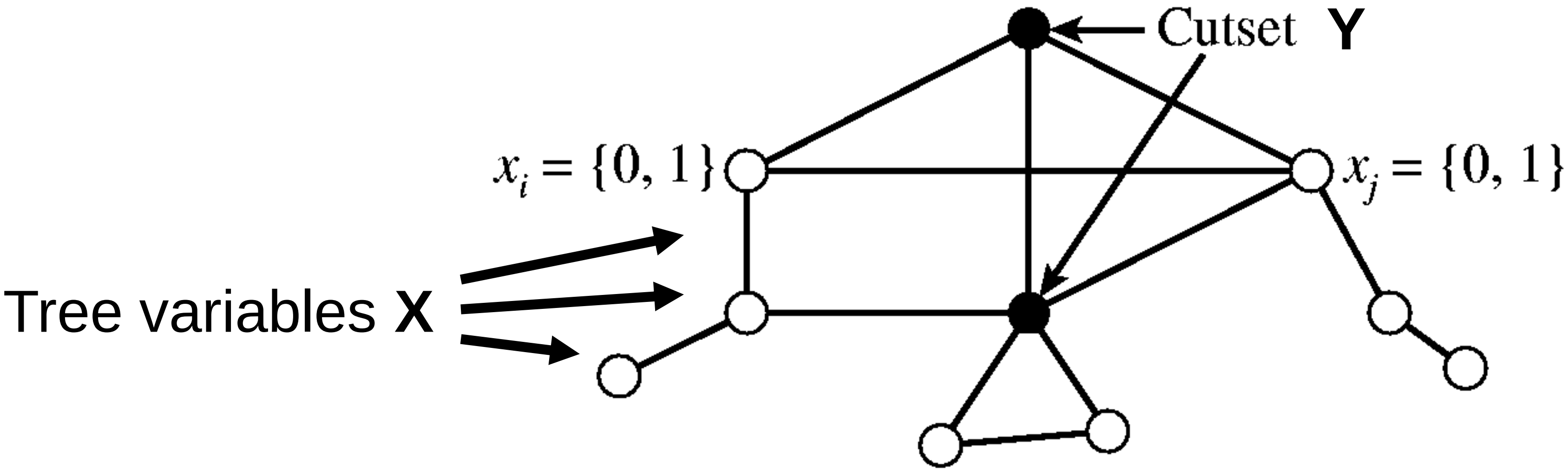
Background:

- Tree algorithm is tractable for trees.
- Networks with bounded width are tractable*.

Basic Scheme:

- Identify a cutset such that width is reduced to desired value.
- Use search with cutset conditioning.

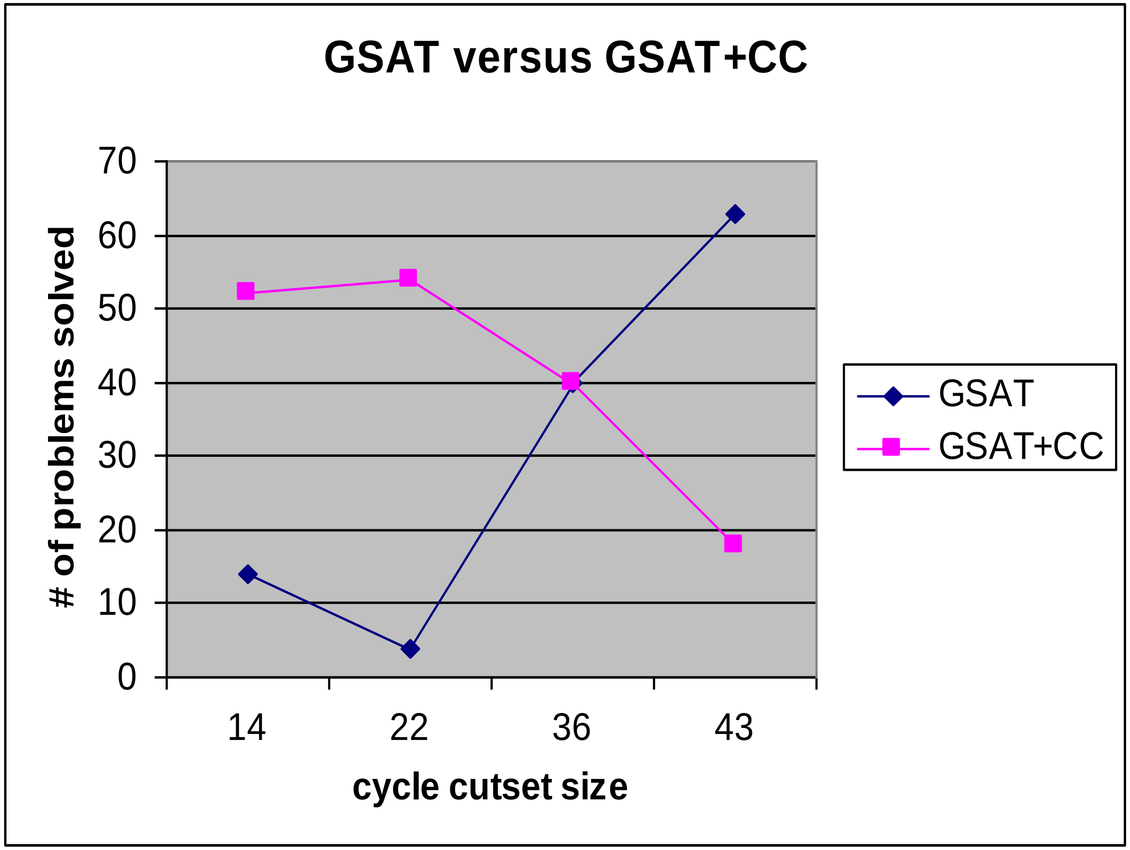
Local search on Cycle-cutset



$$C_{min} = \min_{Y=y} C(y) = \min_{Y=y} \min_{X=x} \{C(X \mid Y = y)\}$$

Results GSAT with Cycle-Cutset

(Kask and Dechter, 1996)



SLS and Cutset Conditioning

Summary:

- A new combined algorithm of SLS and inference based on cutset conditioning
- Empirical evaluation on random CSPs
- SLS combined with the tree algorithm is superior to pure SLS when the cutset is small

Possible project

- Program variants of SLS+Inference
  - Use the computed cost on the tree to guide SLS on the cutset. This is applicable to optimization
  - Replace the cost computation on the tree with simple arc-consistency or unit resolution
  - Implement the idea for SAT using off-the-shelves code: unit-resolution from minisat, SLS from walksat.
- Other projects: start thinking.

More project ideas

- Combine local search with constraint propagation.
- **Pinkas, G., and Dechter, R.,** "On Improving Connectionist Energy Minimization." *In "Journal of Artificial Intelligence Research" (JAIR),* Vol. 3, 1995, pp. 223-248. http://www.ics.uci.edu/%7Ecsp/r24.pdf
- **Kask, K., and Dechter, R.,** "GSAT and Local Consistency." *In "International Joint Conference on Artificial Intelligence" (IJCAI-95), Montreal, Canada, August 1995, pp. 616-622.* http://www.ics.uci.edu/%7Ecsp/r34-gsat-local-consistency.pdf
- **Kask, K., and Dechter, R.,** "Graph-based methods for improving GSAT." *In proceedings of "National Conference of Artificial Intelligence" (AAAI-96), Portland, Oregon, August 1996*
- http://www.ics.uci.edu/%7Ecsp/R46.pdf
- Look at REES