Set 3: Informed Heuristic Search

ICS 271 Fall 2012

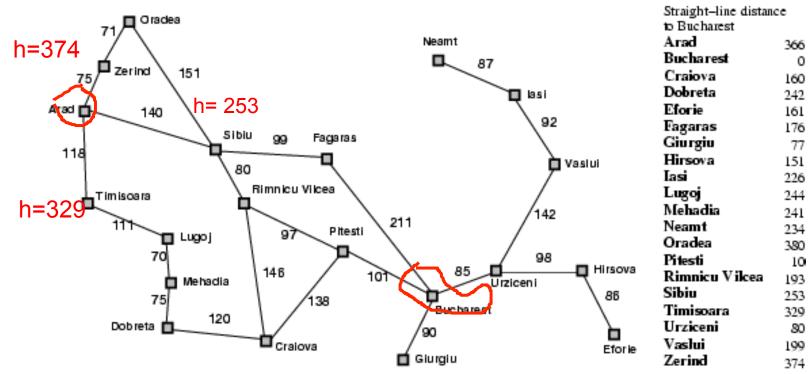
Overview

- Heuristics and Optimal search strategies
 - heuristics
 - hill-climbing algorithms
 - Best-First search
 - A*: optimal search using heuristics
 - Properties of A*
 - admissibility,
 - consistency,
 - accuracy and dominance
 - Optimal efficiency of A*
 - Branch and Bound
 - Iterative deepening A*
 - Automatic generation of heuristics



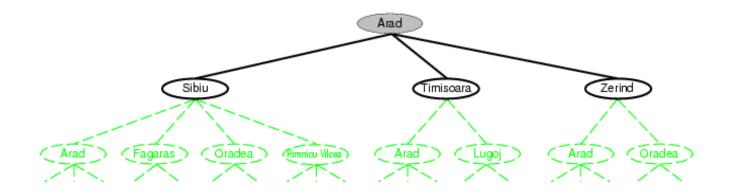
Heuristic Search

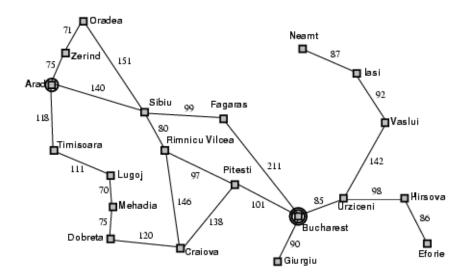
- State-Space Search: every problem is like search of a map
- A problem solving robot finds a path in a state-space graph from start state to goal state, using heuristics



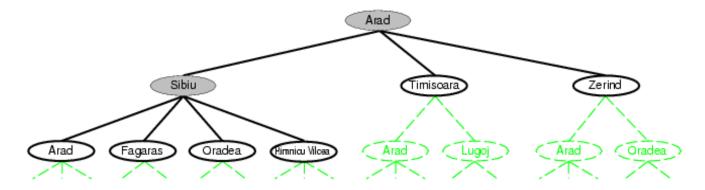
Heuristic = air distance

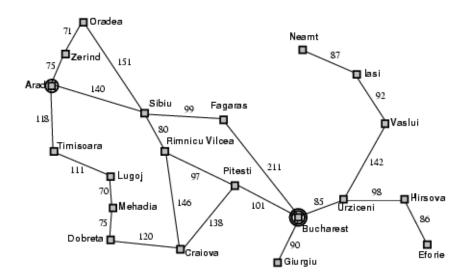
State Space for Path Finding in a Map



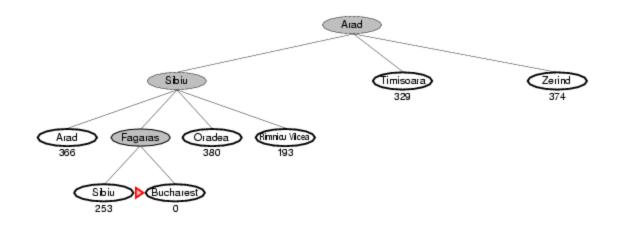


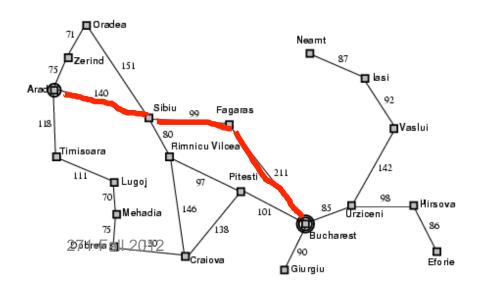
State Space for Path Finding in a Map



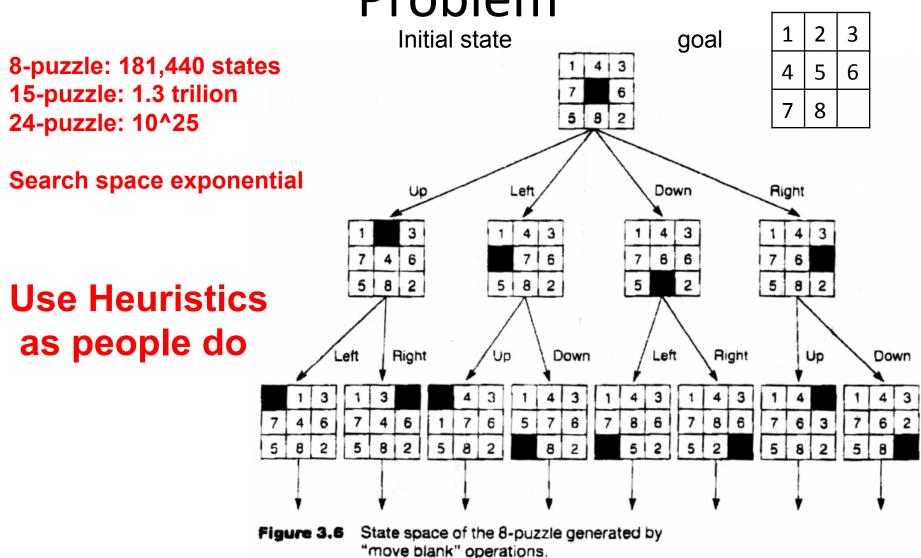


Greedy Search Example

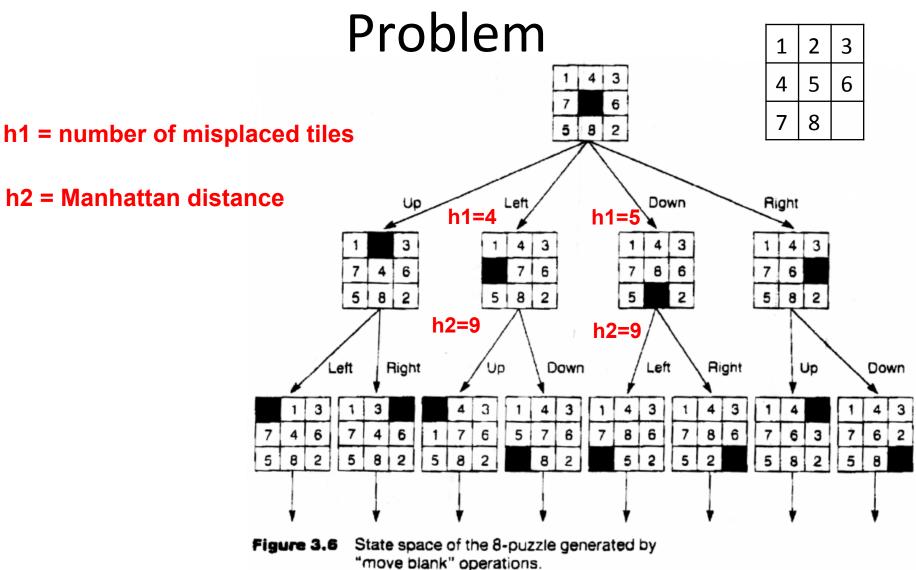




State Space of the 8 Puzzle Problem



State Space of the 8 Puzzle

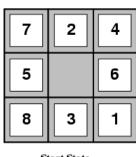


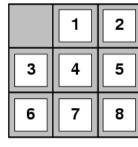
What are Heuristics

- Rule of thumb, intuition
- A quick way to estimate how close we are to the goal. How close is a state to the goal..
- Pearl: "the ever-amazing observation of how much people can accomplish with that simplistic, unreliable information source known as *intuition*."

8-puzzle

- h1(n): number of misplaced tiles
- h2(n): Manhattan distance



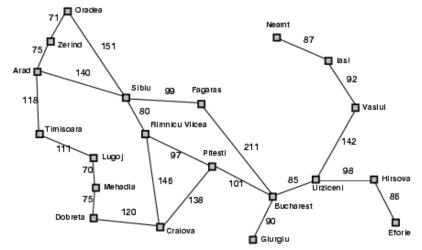


Start State

Goal State

$$h_1(S) = ?$$
 8
 $h_2(S) = ?$ 3+1+2+2+3+3+2 = 18

- Path-finding on a map
 - Euclidean distance



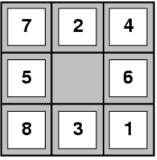
Straight-line distance	
to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Problem: Finding a Minimum Cost Path

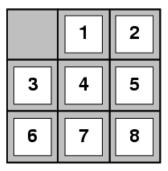
- Previously we wanted an arbitrary path to a goal or best cost. Now, we want the minimum cost path to a goal G
 - Cost of a path = sum of individual transitions along path
- Examples of path-cost:
 - Navigation
 - path-cost = distance to node in miles
 - minimum => minimum time, least fuel
 - VLSI Design
 - path-cost = length of wires between chips
 - minimum => least clock/signal delay
 - 8-Puzzle
 - path-cost = number of pieces moved
 - minimum => least time to solve the puzzle
- Algorithm: Uniform-cost search... still somewhat

Heuristic Functions

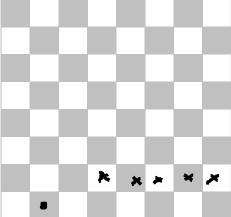
- 8-puzzle
 - Number of misplaced tiles
 - Manhatten distance
 - Gaschnig's
- 8-queen
 - Number of future feasible slots
 - Min number of feasible slots in a row
 - Min number of conflicts (in complete assignments states)
- Travelling salesperson
 - Minimum spanning tree
 - Minimum assignment problem

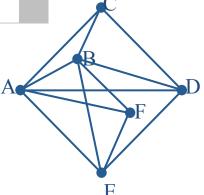






Goal State



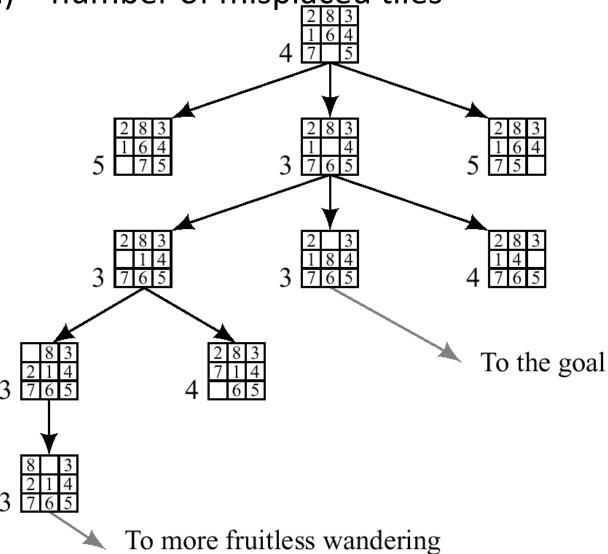


Best-First (Greedy) Search: f(n) = number of misplaced tiles

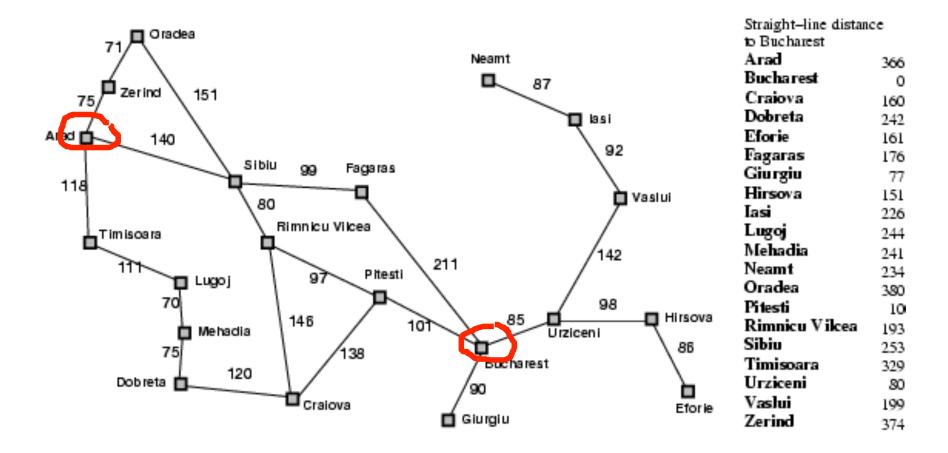


Figure 8.1

Start and Goal Configurations for the Eight-Puzzle



Romania with Step Costs in km



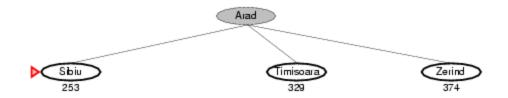
Greedy Best-First Search

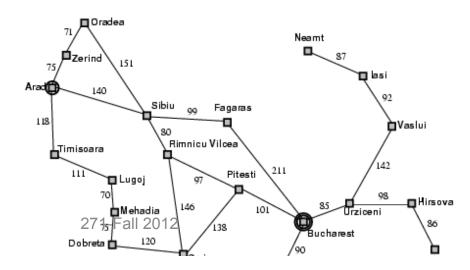
- Evaluation function f(n) = h(n) (heuristic)
- = estimate of cost from n to goal

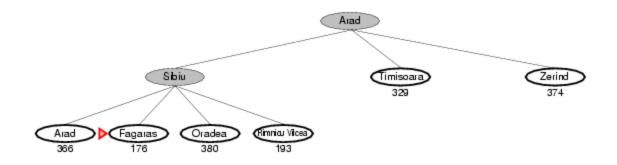
• e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

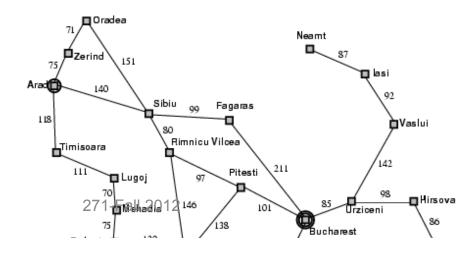
 Greedy best-first search expands the node that appears to be closest to goal

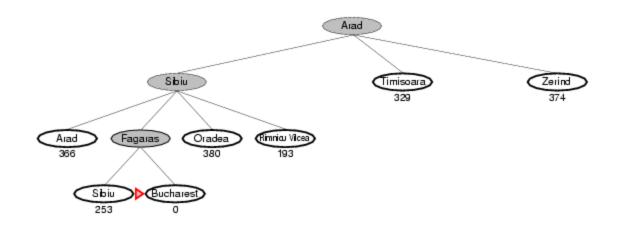


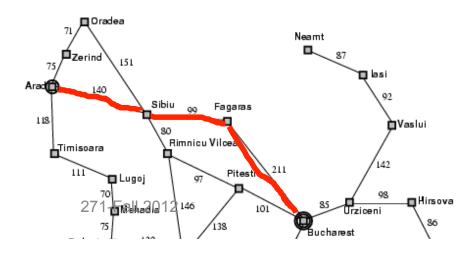












Problems with Greedy Search

- Not complete
- Get stuck on local minimas and plateaus,
- Irrevocable,
- Infinite loops
- Can we incorporate heuristics in systematic search?

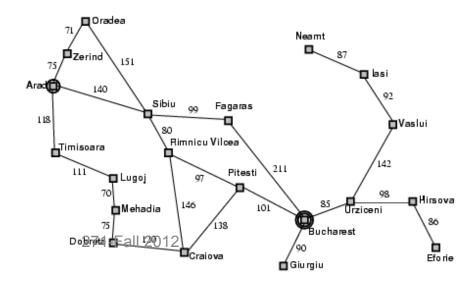
Informed Search - Heuristic Search

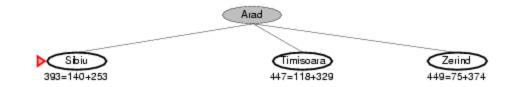
- How to use heuristic knowledge in systematic search?
- Where ? (in node expansion? hill-climbing?)
- Best-first:
 - select the best from all the nodes encountered so far in OPEN.
 - "good" use heuristics
- Heuristic estimates value of a node
 - promise of a node
 - difficulty of solving the subproblem
 - quality of solution represented by node
 - the amount of information gained.
- f(n)- heuristic evaluation function.
 - depends on n, goal, search so far, domain

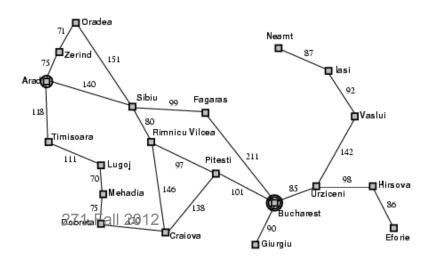
A* Search

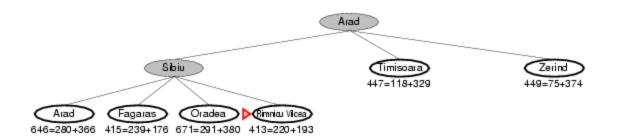
- Idea: avoid expanding paths that are already expensive
- Evaluation function f(n) = g(n) + h(n)
- $g(n) = \cos t$ so far to reach n
- h(n) = estimated cost from n to goal
- f(n) = estimated total cost of path through n to goal

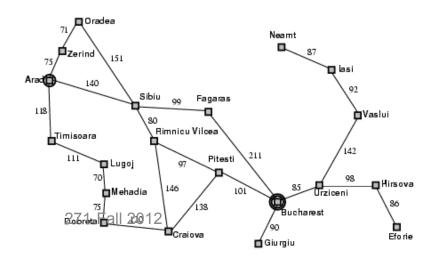


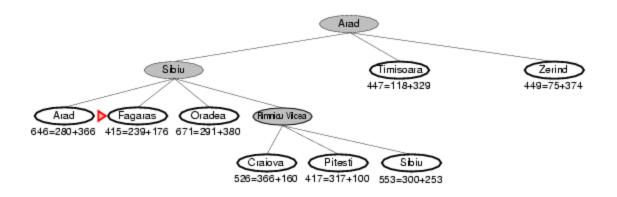


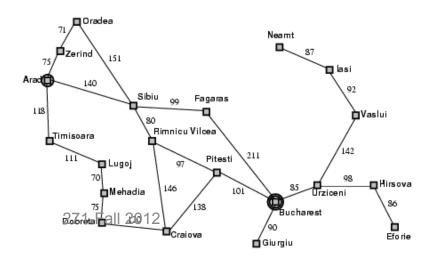


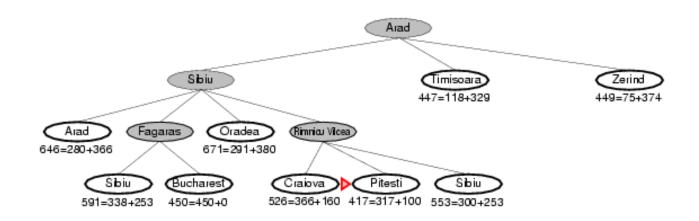


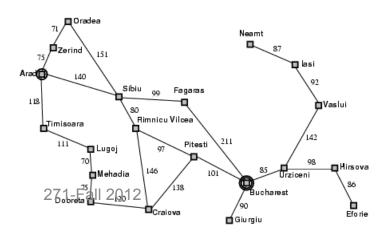


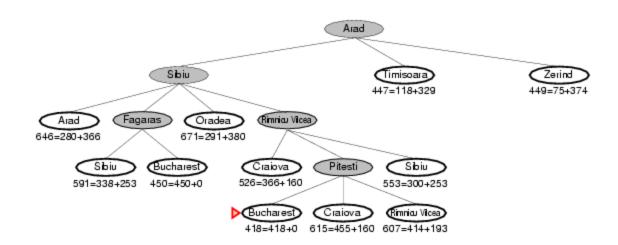








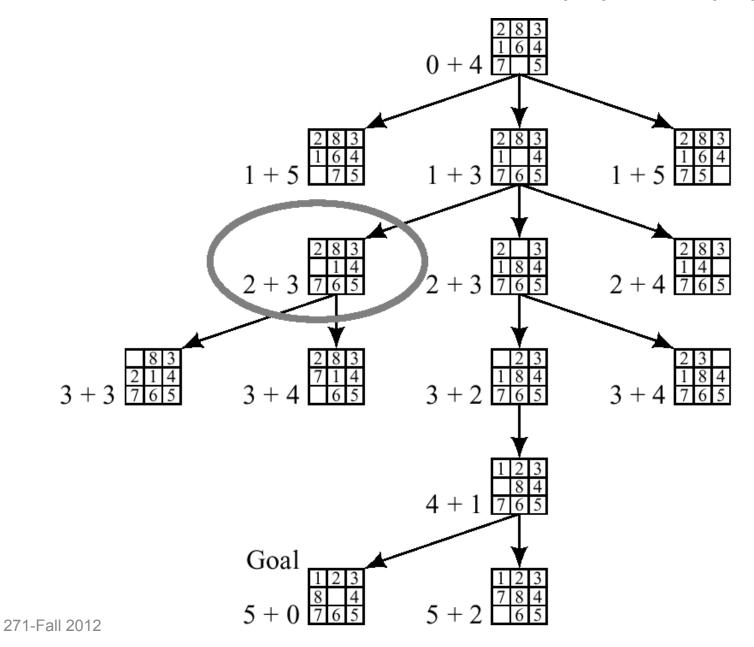




A*- a Special Best-First Search

- Goal: find a minimum sum-cost path
- Notation:
 - c(n,n') cost of arc (n,n')
 - -g(n) = cost of current path from start to node n in the search tree.
 - h(n) = estimate of the cheapest cost of a path from n to a goal.
 - Special evaluation function: f = g+h
- f(n) estimates the cheapest cost solution path that goes through n.
 - h*(n) is the true cheapest cost from n to a goal.
 - g*(n) is the true shortest path from the start s, to n.
- If the heuristic function, h always underestimate the true cost (h(n) is smaller than h*(n)), then A* is guaranteed to find an optimal solution.

A^* on 8-Puzzle with h(n) = w(n)



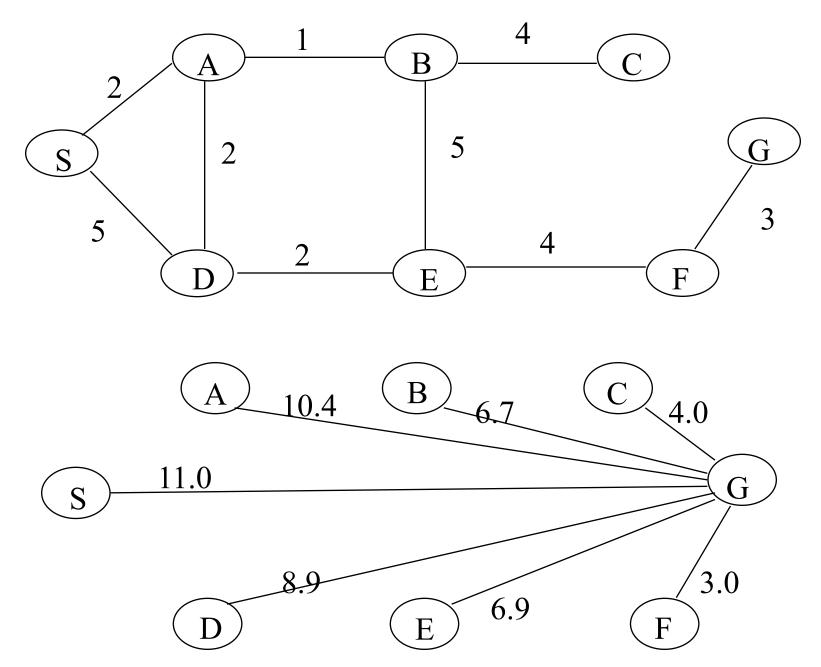
Algorithm A* (with any h on search Graph)

- Input: an implicit search graph problem with cost on the arcs
- Output: the minimal cost path from start node to a goal node.
 - 1. Put the start node s on OPEN.
 - 2. If OPEN is empty, exit with failure
 - 3. Remove from OPEN and place on CLOSED a node n having minimum f.
 - 4. If n is a goal node exit successfully with a solution path obtained by tracing back the pointers from n to s.
 - 5. Otherwise, expand n generating its children and directing pointers from each child node to n.
 - For every child node n' do
 - evaluate h(n') and compute f(n') = g(n') + h(n') = g(n) + c(n,n') + h(n)
 - If n' is already on OPEN or CLOSED compare its new f with the old f. If th enew value is higher, discard the node.
 - Else, put n' with its f value in the right order in OPEN
 - 6. Go to step 2.

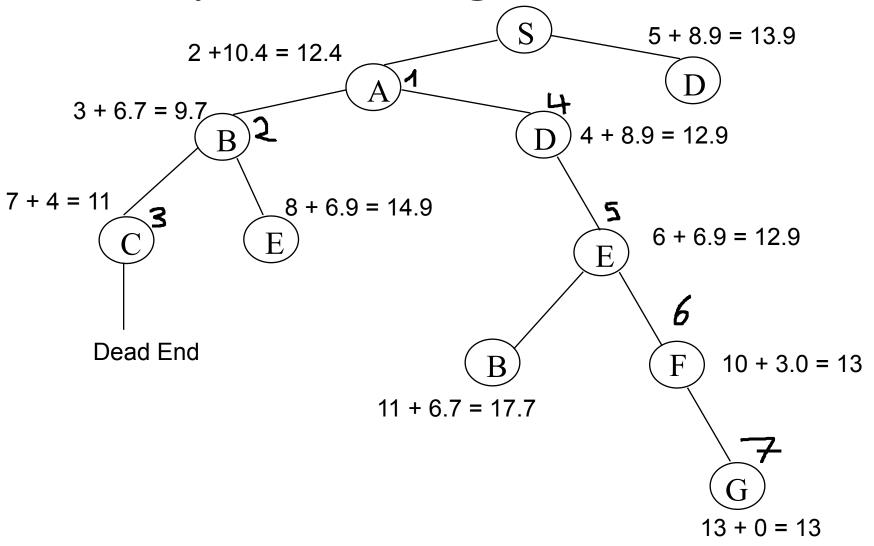
In the book: Uniform search with queue ranked by g+h instead of just g 27\$ [7] but not exactly the same.

Best-First Algorithm *BF* (*)

- 1. Put the start node s on a list called *OPEN* of unexpanded nodes.
- 2. If *OPEN* is empty exit with failure; no solutions exists.
- 3. Remove the first *OPEN* node *n* at which f is minimum (break ties arbitrarily), and place it on a list called *CLOSED* to be used for expanded nodes.
- 4. Expand node n, generating all it's successors with pointers back to n.
- 5. If any of n' s successors is a goal node, exit successfully with the solution obtained by tracing the path along the pointers from the goal back to s.
- 6. For every successor n' on n:
 - a. Calculate f (n').
 - b. if n' was neither on *OPEN* nor on *CLOSED*, add it to *OPEN*. Attach a pointer from n' back to n. Assign the newly computed f(n') to node n'.
 - c. if n' already resided on *OPEN* or *CLOSED*, compare the newly computed f(n') with the value previously assigned to n'. If the old value is lower, discard the newly generated node. If the new value is lower, substitute it for the old (n') now points back to n instead of to its previous predecessor). If the matching node n' resided on *CLOSED*, move it back to *OPEN*.
- 7. Go to step 2.
- * With tests for duplicate nodes.



Example of A* Algorithm in Action



Behavior of A - Termination

- The heuristic function h(n) is called admissible if h(n) is never larger than h*(n), namely h(n) is always less or equal to true cheapest cost from n to the goal.
- A* is admissible if it uses an admissible heuristic, and h(goal) = 0.
- Theorem (completeness) (Hart, Nillson and Raphael, 1968)
 - A* always terminates with a solution path (h is not necessarily admissible) if
 - costs on arcs are positive, above epsilon
 - branching degree is finite.
- Proof: The evaluation function f of nodes expanded must increase eventually (since paths are longer and more costly) until all the nodes on an optimal path are expanded.

Behavior of A* - Completeness

- Theorem (completeness for optimal solution) (HNL, 1968):
 - If the heuristic function is admissible than A* finds an optimal solution.

• Proof:

- 1. A* will expand only nodes whose f-values are less (or equal) to the optimal cost path C* (f(n) is less-or-equal C*).
- 2. The evaluation function of a goal node along an optimal path equals C*.

Lemma:

 Anytime before A* terminates there exists and OPEN node n' on an optimal path with f(n') <= C*.

Consistent (monotone) Heuristics

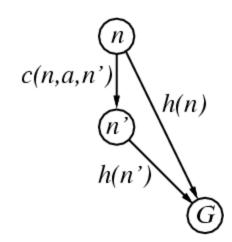
• A heuristic is consistent if for every node n, every successor n' of n generated by any action a,

$$h(n) \le c(n,a,n') + h(n')$$

• If *h* is consistent, we have

$$f(n') = g(n') + h(n')$$

= $g(n) + c(n,a,n') + h(n')$
 $\ge g(n) + h(n)$
= $f(n)$



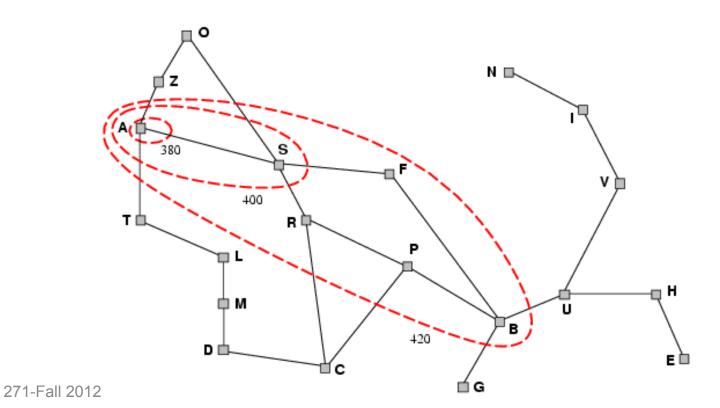
- i.e., f(n) is non-decreasing along any path.
- Theorem: If h(n) is consistent, f along any path is non-decreasing.
- Corollary: the f values seen by A* are non-decreasing.

Consistent Heuristics

- If h is consistent) and h(goal)=0 then h is admissible
 - Proof: (by induction of distance from the goal)
- An A* guided by consistent heuristic finds an optimal paths to all expanded nodes, namely $g(n) = g^*(n)$ for any closed n.
 - Proof: Assume $g(n) > g^*(n)$ and n expanded along a non-optimal path.
 - Let n' be the shallowest OPEN node on optimal path p to n →
 - g(n') = g*(n') and therfore f(n')=g*(n')+h(n')
 - Due to consistency we get $f(n') \le g^*(n') + k(n',n) + h(n)$
 - Since $g^*(n) = g^*(n') + k(n',n)$ along the optimal path, we get that
 - $f(n') \le g^*(n) + h(n)$
 - And since $g(n) > g^*(n)$ then f(n') < g(n) + h(n) = f(n), contradiction

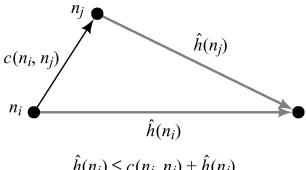
A* with Consistent Heuristics

- A* expands nodes in order of increasing f value
- Gradually adds "f-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Summary of Consistent Heuristics

h is consistent if the heuristic function satisfies triangle inequality for every n and its child node n': h(ni) <= h(nj) + c(ni,nj)



$$\hat{h}(n_i) \le c(n_i, n_j) + \hat{h}(n_j)$$

- When h is consistent, the f values of nodes expanded by A* are never decreasing.
- When A* selected n for expansion it already found the shortest path to it.
- When h is consistent every node is expanded once (if check for duplicates).
- Normally the heuristics we encounter are consistent
 - -the number of misplaced tiles
 - -Manhattan distance
 - –air-line distance

Admissible and Consistent Heuristics?

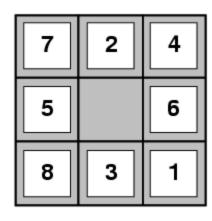
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

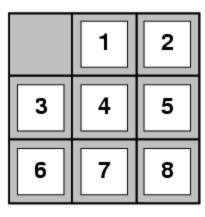
(i.e., no. of squares from desired location of each tile)

The true cost is 26.

Average cost for 8-puzzle is 22. Branching degree 3.







Goal State

•
$$h_1(S) = ?8$$

Effectiveness of A* search

How quality of heuristic impact search?

What is the time and space complexity?

Is any algorithm better? Worse?

Case study: the 8-puzzle

Effectiveness of A* Search Algorithm Average number of nodes expanded

d	IDS	A*(h1)	A*(h2)
2	10	6	6
4	112	13	12
8	6384	39	25
12	364404	227	73
14	3473941	539	113
20	7276	676	

Average over 100 randomly generated 8-puzzle problems h1 = number of tiles in the wrong position h2 = sum of Manhattan distances

Dominance

- Definition: If $h_2(n) \ge h_1(n)$ for all n (both admissible) then h_2 dominates h_1
- *Is h*₂ better for search?
- Typical search costs (average number of nodes expanded):

```
    d=12 IDS = 3,644,035 nodes
        A*(h<sub>1</sub>) = 227 nodes
        A*(h<sub>2</sub>) = 73 nodes
    d=24
        IDS = too many nodes
        A*(h<sub>1</sub>) = 39,135 nodes
        A*(h<sub>2</sub>) = 1,641 nodes
```

Heuristic's Dominance and Pruning Power

- Definition:
 - A heuristic function h (strictly) dominates h' if both are admissible and for every node n, h(n) is (strictly) greater than h' (n).
- Theorem (Hart, Nillson and Raphale, 1968):
 - An A* search with a dominating heuristic function h has the property that any node it expands is also expanded by A* with h'.
- Question: Does manhattan distance dominate the number of misplaced tiles?
- Extreme cases
 - h = 0
 - -h=h*

Summary of A* properties

- A* expands every path along which f(n) < C*
- A* will never expand any node s.t. f(n) > C*
- If h is consistent A* will expand any node such that f(n)
- Therefore, A* expands all the nodes for which f(n) < C* and a subset of the nodes for which f(n) = C*.
- Therefore, if h₁(n) < h₂(n) clearly the subset of nodes expanded by h_2 is smaller.

Non-admissible heuristics: Adjust weights of g and h

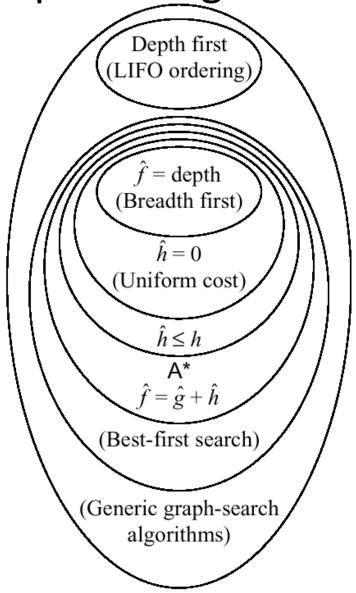
$$f_w(n) = (1 - w)g(n) + w \cdot h(n)$$

- W = 0 (uniform cost)
- W=1/2 (A*)
- W=1 (DFS greedy)
- If h is admissible then f_w is admissible for 0 <= w <= 1/2

Complexity of A*

- A* is optimally efficient (Dechter and Pearl 1985):
 - It can be shown that all algorithms that do not expand a node which A* did expand (inside the contours) may miss an optimal solution
- A* worst-case time complexity:
 - is exponential unless the heuristic function is very accurate
- If h is exact (h = h*)
 - search focus only on optimal paths
- Main problem: space complexity is exponential
- Effective branching factor:
 - logarithm of base (d+1) of average number of nodes expanded.

Relationships among Search Algorithms

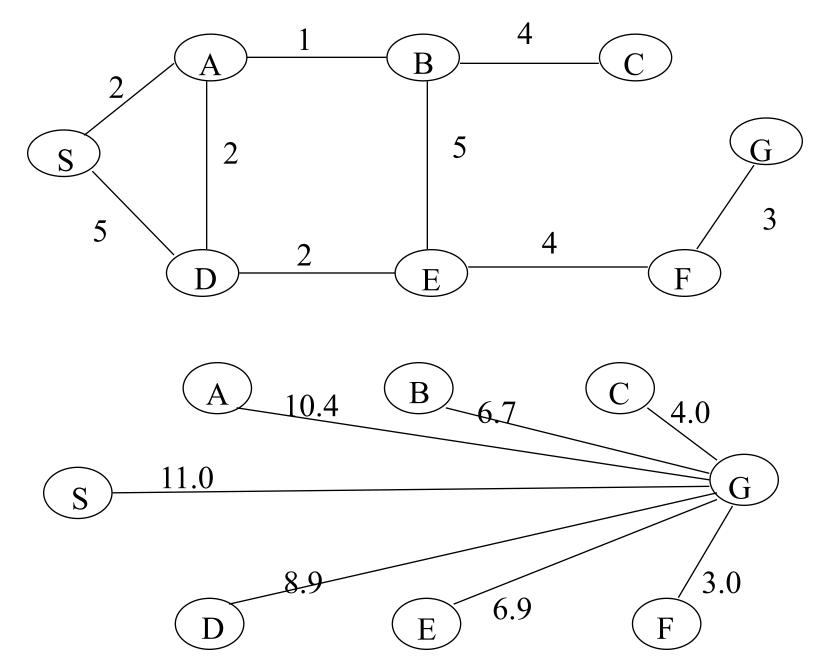


Pseudocode for Branch and Bound Search (An informed depth-first search)

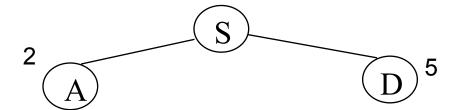
```
Initialize: Let Q = \{S\}
While Q is not empty
         pull Q1, the first element in Q
         if Q1 is a goal compute the cost of the solution and update
            L <-- minimum between new cost and old cost
         else
                   child nodes = expand(Q1),
                   <eliminate child nodes which represent simple
                   loops>,
                   For each child node n do:
                              evaluate f(n). If f(n) is greater than L
                              discard n
                   end-for
                   Put remaining child nodes on top of queue
                   in the order of their evaluation function, f.
```

end

Continue



Example of Branch and Bound in action

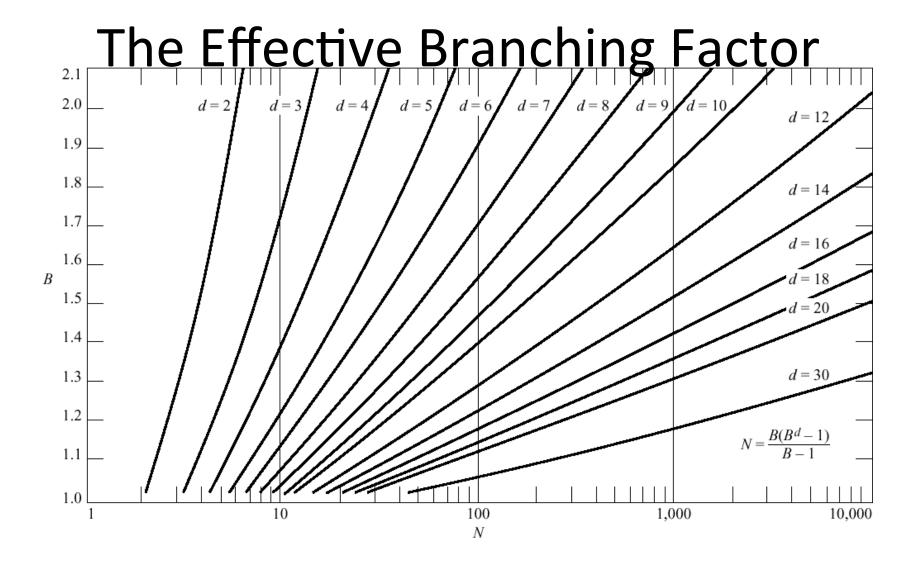


Properties of Branch-and-Bound

- Not guaranteed to terminate unless has depth-bound
- Optimal:
 - finds an optimal solution
- Time complexity: exponential
- Space complexity: can be linear

Iterative Deepening A* (IDA*) (combining Branch-and-Bound and A*)

- Initialize: f <-- the evaluation function of the start node
- until goal node is found
 - Loop:
 - Do Branch-and-bound with upper-bound L equal current evaluation function f.
 - Increment evaluation function to next contour level
 - end
- continue
- Properties:
 - Guarantee to find an optimal solution
 - time: exponential, like A*
 - space: linear, like B&B.
 - Problems: The number of iterations may be large.



Inventing Heuristics automatically

- Examples of Heuristic Functions for A*
 - the 8-puzzle problem
 - the number of tiles in the wrong position
 - is this admissible?
 - Manhattan distance
 - is this admissible?
 - How can we invent admissible heuristics in general?
 - look at "relaxed" problem where constraints are removed
 - e.g.., we can move in straight lines between cities
 - e.g.., we can move tiles independently of each other

Inventing Heuristics Automatically (continued)

- How did we
 - find h1 and h2 for the 8-puzzle?
 - verify admissibility?
 - prove that air-distance is admissible? MST admissible?
- Hypothetical answer:
 - Heuristic are generated from relaxed problems
 - Hypothesis: relaxed problems are easier to solve
- In relaxed models the search space has more operators, or more directed arcs
- Example: 8 puzzle:
 - A tile can be moved from A to B if A is adjacent to B and B is clear
 - We can generate relaxed problems by removing one or more of the conditions
 - A tile can be moved from A to B if A is adjacent to B
 - ...if B is blank
 - A tile can be moved from A to B.

Relaxed Problems

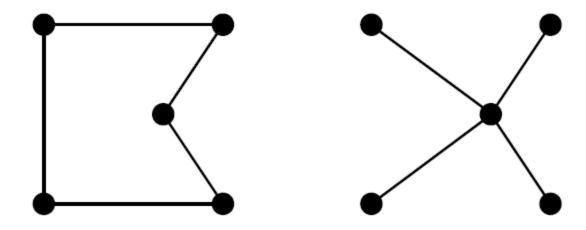
- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ (number of misplaced tiles) gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ (Manhatten distance) gives the shortest solution

Generating heuristics (continued)

- Example: TSP
- Find a tour. A tour is:
 - 1. A graph
 - 2. Connected
 - 3. Each node has degree 2.
- Eliminating 3 yields MST.

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP) Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour

Automating Heuristic generation

- Use STRIPs language representation:
- Operators:
 - pre-conditions, add-list, delete list
- 8-puzzle example:
 - on(x,y), clear(y) adj(y,z) ,tiles x1,...,x8
- States: conjunction of predicates:
 - on(x1,c1),on(x2,c2)....on(x8,c8),clear(c9)
- move(x,c1,c2) (move tile x from location c1 to location c2)
 - pre-cond: on(x1,c1), clear(c2), adj(c1,c2)
 - add-list: on(x1,c2), clear(c1)
 - delete-list: on(x1,c1), clear(c2)
- Relaxation:
- 1. Remove from prec-cond: clear(c2), adj(c2,c3) → #misplaced tiles
- 2. Remove clear(c2) → manhatten distance
- 3. Remove adj(c2,c3) → h3, a new procedure that transfer to the empty location a tile appearing there in the goal

Heuristic generation

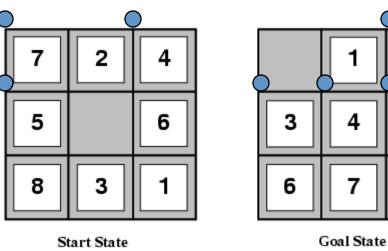
- The space of relaxations can be enriched by predicate refinements
- adj(y,z) iff neigbour(y,z) and same-line(y,z)
- Theorem: Heuristics that are generated from relaxed models are consistent.
- Proof: h is true shortest path in a relaxed model
 - h(n) <=c'(n,n')+h(n') (c' are shortest distances in relaxed graph)</p>
 - c'(n,n') <= c(n,n')
 - $\rightarrow h(n) \le c(n,n') + h(n')$
- Problem: not every relaxed problem is easy, often, a simpler problem which is more constrained will provide a good upperbound.
- The main question: how to recognize a relaxed easy problem.
- A proposal: a problem is easy if it can be solved optimally by a greedy algorithm

Improving Heuristics

- If we have several heuristics which are non dominating we can select the max value.
- Reinforcement learning.

• Pattern Databases: you can solve optimally a

sub-problem



8

Pattern Databases

- For sliding tiles and Rubic's cube
- For a subset of the tiles compute shortest path to the goal using breadthfirst search
- For 15 puzzles, if we have 7 fringe tiles and one blank, the number of patterns to store are 16!/(16-8)! = 518,918,400.
- For each table entry we store the shortest number of moves to the goal from the current location.
- Use different subsets of tiles and take the max heuristic during IDA* search. The number of nodes to solve 15 puzzles was reduced by a factor of 346 (Culberson and Schaeffer)
- How can this be genaralized? (a possible project)

Problem-reduction representations AND/OR search spaces

Decomposable production systems (Natural language parsing)

Initial database: (C,B,Z)

Rules: R1: C \rightarrow (D,L)

R2: $C \rightarrow (B,M)$

R3: $B \rightarrow (M,M)$

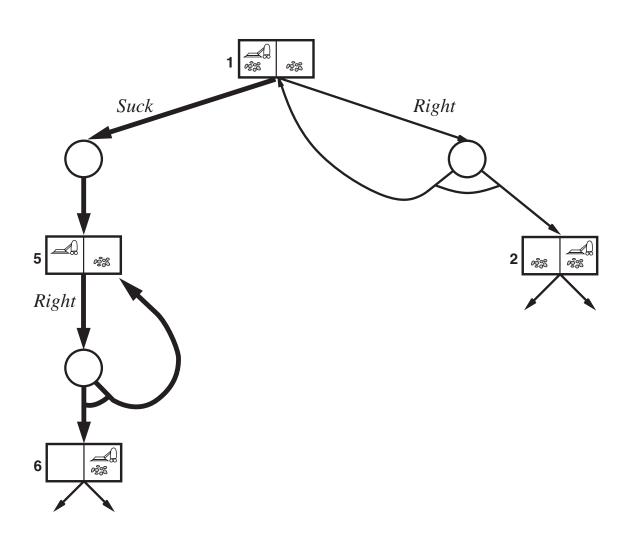
R4: $Z \rightarrow (B,B,M)$

Find a path generating a string with M's only.

- The erratic vacuum world (actions are non-deterministic)
- The tower of Hanoi

To move n disks from peg 1 to peg 3 using peg 2 Move n-1 pegs to peg 2 via peg 3, move the nth disk to peg 3, move n-1 disks from peg 2 to peg 3 via peg 1.

Erratic Vacuum



AND/OR Graphs

- Nodes represent subproblems
 - And links represent subproblem decompositions
 - OR links represent alternative solutions
 - Start node is initial problem
 - Terminal nodes are solved subproblems
- Solution graph
 - It is an AND/OR subgraph such that:
 - 1. It contains the start node
 - 2. All it terminal nodes (nodes with no successors) are solved primitive problems
 - 3. If it contains an AND node L, it must contain the entire group of AND links that leads to children of L.

Algorithms searching AND/OR graphs

- All algorithms generalize using hyper-arc suscessors rather than simple arcs.
- AO*: is A* that searches AND/OR graphs for a solution subgraph.
- The cost of a solution graph is the sum cost of it arcs. It can be defined recursively as: k(n,N) = c_n+k(n1,N)+...k(n_k,N)
- h*(n) is the cost of an optimal solution graph from n to a set of goal nodes
- h(n) is an admissible heuristic for h*(n)
- Monotonicity:
- h(n)<= c+h(n1)+...h(nk) where n1,...nk are successors of n
- AO* is guaranteed to find an optimal solution when it terminates if the heuristic function is admissiblels h is

Summary

- In practice we often want the goal with the minimum cost path
- Exhaustive search is impractical except on small problems
- Heuristic estimates of the path cost from a node to the goal can be efficient in reducing the search space.
- The A* algorithm combines all of these ideas with admissible heuristics (which underestimate), guaranteeing optimality.
- Properties of heuristics:
 - admissibility, consistency, dominance, accuracy
- Reading
 - R&N Chapter 3-4

Beyond Classical Search (chapter 4 3rd eddition)

- Local search for optimization
 - Greedy, hill-climbing search, simulated annealing, local beem search, genetic algorithms
 - Local search in continuous spaces
- Searching with non-deterministic actions
 - The erratic vaccum cleaner example
 - Using and/or search spaces.
- Searching with partial observations
 - Using belief states
- Online search agents and unknown environments
 - Actions, costs, goal-tests are revealed in state only
 - Exploration problems. Safely explorable